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Journal
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ON REFLEX VISUAL SENSATIONS*

By Frank Allen

Synopsis

Section I of this communication contains a very brief summary of previous work of the author on the application of the method of the critical frequency of flicker to the determination of the primary color sensations. Attention is specially directed to the fact that former investigations were conducted in a dark room, so that while the right eye was stimulated by light from the spectrum, the left eye was in darkness adaptation. It was generally believed that darkness adaptation of one eye had no influence upon the other.

Section II contains descriptions of experiments made in a room illuminated by ordinary daylight. The right eye was in daylight adaptation while the left eye was fatigued with spectral lights. All measurements of the critical frequency of flicker were made with the right eye. It was found that the stimulus applied to the left eye was transferred to the right eye by some reflex process resulting in an enhancement of the brightness of three colors, which were always red, green, and violet. Six colors were found, viz., .660μ, .570μ, .520μ, .505μ, .480μ and .425μ, which produced no effect. The reflex and normal curves always coincided in two places which averaged about .653μ and .50μ. The magnitude of the reflex effect is greatest for the violet color, and seems to vary as some inverse function of the wave length.

Section III contains the description of experiments performed when the right eye was fatigued and the left was always in daylight adaptation.

It was found that fatigue produced both a direct and a reflex effect. The red, green, and violet colors showed one elevation of the fatigue curve above the normal in the part corresponding to the color used, and two reflex depressions in parts corresponding to the remaining primary colors. The compound colors, yellow and blue, produced two elevations in the parts of the curves corresponding to the color sensations of which they are compounded, and one reflex depression in the part corresponding to the remaining color. The number of elevations or depressions in each curve was always three, corresponding to red, green, and violet.

The six colors mentioned above produced no effect, and are termed equilibrium colors.

The effect of darkness adaptation of one eye upon the other is shown to cause a diminution of brightness of the whole spectrum. This is applied to explain Fechner's paradox.

Section IV contains theoretical applications to explain the cause of the whiteness underlying all color sensations, successive contrast, binocular contrast, non-saturation of colors,

* Read at the Meeting of the Royal Society of Canada, Ottawa, May, 1922. Also read at the Meeting of the Optical Society of America, Washington, D.C., Oct. 27th, 1922.

saturation of colors in after images, the duplicity theory of von Kries, complementary colors, and to theories of color vision.

Section V contains anatomical and physiological considerations which render it probable that the effects discovered are due to a new kind of physiological reflex which is termed a sensory reflex.

I.

In former communications[1] to the Physical Review and the Philosophical Magazine, studies were made of the effect of fatiguing the eye with various spectral hues on the color sensations. The method used was to determine the persistence of vision, or, as it is less euphoniously but more exactly termed, the critical frequency of flicker, of successive colors of the spectrum, when the eye was first in its normal and then in a fatigued condition. The two sets of measurements were expressed as persistency curves, of which many new examples are shown in this communication. The coincidences and divergences of the curves enabled conclusions to be drawn regarding the number of the fundamental color sensations, and the limits of the hues of the spectrum by which they are excited. It was found that the divergences of the curves, in the form of elevations above the normal, occurred only in three portions which correspond to the red, green and violet regions of the spectrum. Each of these three colors caused one elevation in the part of the spectrum corresponding to itself; but the remaining colors each caused two elevations which were pairs of those obtained with simple colors. It was found that when the retina was fatigued with the four colors, .660μ, .570μ, .470μ, and .420μ, the normal and fatigue curves coincided, and these transition points were believed to mark the boundaries between the simple and compound colors.

From the point of view of the present investigation, it is most important to bear in mind the conditions under which the former curves were obtained. The Nichols Apparatus,[2] as it may be termed, consisting of a constant acetylene flame, a rotating sectored disk, and a spectrometer, with suitable electric recording devices for measuring the speed of the disk, was set up in a dark room. The normal curve was made with the right eye when both eyes were in daylight adaptation. The fatigue curves were obtained while the observer stayed in the dark room continuously for from two to three hours. Thus the

[1] Phil. Mag. *38*, p. 56; 1919.
Phys. Rev. *11*, p. 257; 1900.

[2] E. L. Nichols, Am. Jour. Sci. *28*; 1884. Also, Allen, Phil. Mag. loc. cit. and Phys. Rev. loc. cit.

left eye of the observer was in continuous darkness adaptation while the right eye was being fatigued. When these curves were being obtained, it was supposed that the simplest condition of the eye prevailed. It was not suspected in the least that darkness adaptation of one eye affected the other. It must not, however, be inferred from the new curves about to be considered that the old are wrong, but only that they represent the combined effect of fatigue of one eye and the transferred effect of darkness adaptation of the other.

II. REFLEX CURVES

The experiments now to be described differ from those just referred to, in two important particulars. First, the traditional dark-room, inherited from Newton, was abandoned, and the apparatus transferred to a room well illuminated by ordinary daylight. Second, the left eye only was maintained in a state of fatigue and the critical time of flicker was measured by the right eye which was constantly in daylight adaptation. The instruments were properly screened from all extraneous light, and a shield, fitted to the eye, was attached to the eyepiece of the observing spectrometer, which enabled all measurements to be made under exactly the same conditions. So constant were the acetylene flame, which supplied the observing spectrum, the position and adjustments of all apparatus, and notably the retina in its ordinary condition, that the normal curve remained the same for the five months during which these researches were in progress. The use of a lighted room also enabled the work to be done under far more agreeable physical conditions, and the curves are much more fully and exactly determined than formerly.

The spectrum used for fatigue was obtained from an arc light with a two-prism Browning spectrometer, and was so intense that the colors in the middle approached whiteness in appearance. A narrow but quite long rectangular portion was isolated with a shutter eyepiece and the wave lengths mentioned in this paper are for the centre of this band. The measurements were made upon a much shorter and slightly narrower band in the eyepiece of a four-prism Hilger spectrometer. The centre of the retina in each eye alone was used.

In making observations, an invariable procedure was followed. First, all instruments were placed in adjustment and in readiness for recording the speed of the disk; second, the eye was exposed to the fatiguing light for from two to three minutes; and, finally, the speed of the disk was adjusted for the critical frequency of flicker and recorded. Only

one fatiguing color was used on any day, except for some partial curves. The time required to obtain all the measurements for a curve was about three hours, sometimes employed continuously, sometimes divided into two portions. At least two independent chronographic records, each giving several readings, were made for each point of a curve. The values used in plotting were usually means of six, except for the ends of the spectrum. The character of the curves was determined by the first set of readings; but for the sake of smoothness, which is the obvious characteristic of the spectrum, additional readings were taken for all doubtful or apparently misplaced points. As all conditions were constant there was no limit to the number of determinations which might be made. In each figure, two curves are plotted, one, the normal or reference curve, which throughout is uniformly shown as a dashed line, the second, that which is obtained with the eye in some abnormal condition. With no exception, all curves described in this paper were made with the right eye of the author.

The two curves are plotted together to the same scale, and by their coincidences and divergences show the effect of the color stimulus under study, as far as it can be shown by this method.

At the bottom of each figure the same curves are reproduced in another form by giving the normal curve an arbitrary value of 100 in all parts, and reducing the other curve in the same proportion. This enables the position and extent of the divergences to be readily seen. The position of the fatiguing color is indicated by a short, vertical line.

The retina is not very sensitive to small changes in the brightness of colors when directly observed. But a variation in brightness that changes the mean value of the critical frequency of flicker by 0.0002 second can quite confidently be measured for the brighter colors in the middle of the spectrum. It is known[3] "that the point of fusion of intermittent stimuli, so as to produce a continuous sensation, depends, not on the physical intensities of the stimuli, but on their physiological intensities, as determined by the condition and nature of the stimulated retina." It has also been proved by Ferry[4] and by T. C. Porter[5] "that the duration of the undiminished sensation produced by different spectral hues depends solely on the luminosity of the colors and not on their wave frequency."[6] We are justified therefore in inter-

[3] Quoted from Rivers in Parsons, "Colour Vision," p. 95.
[4] Am. Jour. Sci. *3*, p. 44; 1892.
[5] Proc. Roy. Soc. *63;* 1898.
[6] Parsons, loc. cit. p. 96.

preting an elevation of the abnormal curve above the normal as a decrease in the physiological brightness of the corresponding part of the spectrum; a depression below the normal as an increase in the brightness; and coincidence of them as indicating no change whatever.

At the outset it was a matter of conjecture what changes would occur to one eye when the other was fatigued. The character of the curves could not in the least be anticipated. That certain color and light effects are transferred from one eye to the other is well known. An observation of Burch's[7] which the author happened upon during the progress of the investigation, is connected with this subject and has possibly some practical applications in connection with the use of the microscope and telescope. Burch observed that when one eye is illuminated by bright white light, the scintillations in a spinthariscope are rendered much more brilliant to the other eye which has meanwhile been kept in darkness.

The curves were not obtained in the systematic order in which they are here presented, and the precise colors used were selected for reasons which presented themselves as the work progressed. The data for all the curves discussed in this section are given together in Table 1.

TABLE 1. *Reflex Curves*

λ	Normal	.740μ	.690μ	.687μ	.677μ	.660μ	.625μ	.577μ	.570μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.750μ	0.0360	0.0351	0.0339	0.0337	0.0339	0.0366	0.0332	0.0324	0.0355
.740	.0335	.0328	.0318	.0316	.....	.0329	.0306	.....	.0331
.720	.0287	.0283	.0274	.0272	.0278	.....	.0266	.0265	.0280
.710	.....	.....	.....	.....	.....	.....	.....	.0247	.....
.700	.0244	.0237	.0236	.0238	.0236	.0243	.0230	.0223	.0245
.680	.....	.....	.....	.0202	.0199	.....	.0196	.0186	.0201
.660	.0174	.0172	.0172	.0172	.....	.0173	.0171	.0159	.0174
.640	.....	.0158	.....	.0151	.0156	.....	.0154	.0146	.0159
.620	.0152	.0148	.0147	.0141	.0148	.0154	.0146	.0139	.0151
.590	.0148	.0143	.0139	.0135	.0140	.0147	.0142	.0136	.0145
.550	.0155	.0148	.0146	.0142	.0148	.0152	.0148	.0145	.0153
.530	.0165	.0159	.0157	.0154	.0161	.0164	.0156	.0155	.0164
.520	.....	.....	.0165	.....	.....	.....	.....	.....	.....
.506	.....	.....	.....	.0196	.....	.....	.0190	.0192	.....
.500	.0209	.0201	.0204	.....	.0212	.0207	.0208	.....	.0206
.480	.0255	.....	.0249	.0255	.0247	.0250	.0252	.0253	.0251
.450	.0336	.0325	.0328	.0328	.0321	.0341	.0321	.0326	.0333
.433	.....	.....	.....	.0369	.0363	.....	.....	.....	.....
.420	.0431	.0422	.0408	.0407	.0409	.0431	.0406	.0406	.....
.410	.0459	.0455	.0430	.0433	.0433	.0461	.0435	.0436	.0451

[7] Burch: "Physiological Optics," p. 91.

TABLE 1. *Reflex Curves (continued)*

λ	.550μ	.530μ	.520μ	.515μ	.505μ	.495μ	.480μ	.460μ	.450μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.750μ	0.0322	0.0326	0.0353	0.0353	0.0358	0.0346	0.0354	0.0336	0.0331
.740	.0308	.0306	.0337	.0334	.0333	.0324	.0332	.0316	.0307
.720	.0273	.0266	.0284	.0284	.0282	.0271	.0281	.0273	.0270
.700	.0234	.0230	.0245	.0241	.0241	.0233	.0242	.0235	.0233
.680	.0200	.0197	.....	.....	.....	.....	.....	.0198	.0197
.660	.0172	.0171	.0170	.0176	.0177	.0173	.0173	.0172	.0173
.640	.0158	.0161	.0158	.0161	.0161	.0159	.0160	.0160	.0159
.620	.0151	.0146	.0154	.0153	.0153	.0148	.0151	.0152	.0149
.590	.0144	.0140	.0147	.0144	.0148	.0141	.0148	.0144	.0141
.550	.0148	.0144	.0153	.0147	.0153	.0146	.0153	.0148	.0148
.530	.0159	.0155	.0161	.0156	.0165	.0157	.0163	.0157	.0157
.500	.0205	.0201	.0207	.0202	.0213	.0204	.0206	.0204	.0205
.480	.0243	.0237	.0254	.0238	.0262	.0249	.0256	.0250	.0247
.467	.....	.....	.0294	.0268	.0292	.0278	.0284	.....	.0279
.450	.0318	.0318	.0333	.0318	.0334	.0326	.0342	.0322	.0313
.433	.....	.....	.0391	.0368	.0387	.....	.0381	.0369	.0354
.420	.0390	.0399	.0430	.0407	.0425	.0416	.0432	.0414	.0397
.410	.0421	.0418	.0450	.0436	.0463	.0454	.0455	.0437	.0425

λ	.425μ	.420μ	.410μ	White	Dark-ness				
	Sec.	Sec.	Sec.	Sec.	Sec.				
.750μ	0.0356	0.0348	0.0341	0.0337	0.0390	.....	.....	.....	.....
.740	.0333	.0325	.0319	.0314	.0353	.....	.....	.....	.....
.730	.....	.....	.0296	.....	.....	.....	.....	.....	.....
.720	.0284	.0278	.0271	.0267	.0302	.....	.....	.....	.....
.710	.....	.....	.0254	.....	.....	.....	.....	.....	.....
.700	.0246	.0240	.0234	.0232	.0260	.....	.....	.....	.....
.680	.0196	.0203	.0200	.....	.....	.....	.....	.....	.....
.660	.0173	.0173	.0173	.0164	.0190	.....	.....	.....	.....
.640	.0160	.0161	.0153	.0153	.0173	.....	.....	.....	.....
.620	.0153	.0151	.0140	.0145	.0161	.....	.....	.....	.....
.590	.0148	.0144	.0134	.0138	.0156	.....	.....	.....	.....
.550	.0152	.0151	.0140	.0143	.0163	.....	.....	.....	.....
.530	.0164	.0160	.0153	.0155	.0178	.....	.....	.....	.....
.500	.0205	.0204	.0193	.0194	.0222	.....	.....	.....	.....
.480	.0248	.0260	.0234	.0237	.0271	.....	.....	.....	.....
.467	.0282	.0288	.0259	.....	.....	.....	.....	.....	.....
.450	.0331	.0333	.0306	.0319	.0363	.....	.....	.....	.....
.433	.0392	.....	.0358	.0360	.0433	.....	.....	.....	.....
.420	.0430	.0422	.0400	.....	.....	.....	.....	.....	.....
.410	.0457	.0456	.0432	.0426	.0499	.....	.....	.....	.....

The first curve to be discussed was nearly the last to be secured. The left eye was fatigued with the red light of wave length .740μ, as bright as it could be obtained in the arc spectrum employed. The curve is shown in Fig. 1. There are three slight depressions of the abnormal curve below the normal, one occurring at each end, corre-

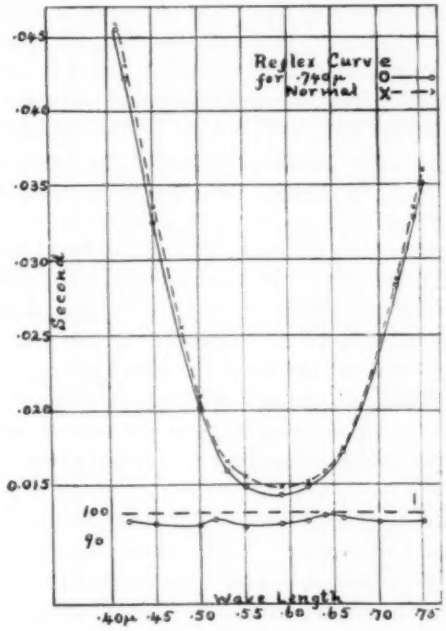


FIG. 1

sponding to the red and violet regions of the spectrum, and the third in the middle corresponding to the green. The depressions are not confined to the precise limits of these colors but extend into the surrounding regions. The conclusion is at once evident that the effect of fatigue of the left eye has been transferred to the right eye by some reciprocal or reflex visual process, and there results in an enhancement of the brightness of three spectral regions. The three depressions are separated by two coincidence points at about .64μ and .52μ, where the normal and abnormal curves coincide, or approach each other closely. The curve obtained when the fatiguing color was .690μ is shown in Fig. 2. The depressions are greatly deepened indicating that the reflex effect of this color is much greater than that of the

Note. In most, and perhaps all, of the curves the greatest enhancement is complementary to the fatiguing color.

preceding. In Fig. 3 is shown the reflex curve for .687μ, which exhibits the maximum effect for red colors. The curve for the reflex effect of .677μ in Fig. 4 shows a diminution of the amount of enhancement of the brightness. These four curves are of the same type, but in addition to the varying magnitude of the reflex effects they show that the two coincidences of the curves do not occur at exactly the same points. We may conclude, however, that there are always two coincidences which occur near the wave lengths, .66μ and .50μ.

It now became of great interest to ascertain whether the transition colors would exhibit any reflex effects. The transition color .660μ was employed as the fatiguing stimulus with the result shown in Fig. 5. The curves coincide throughout their length showing that the brightness of the spectrum appears the same as when the eye is normal.

The narrowness of this peculiar region is well brought out by the fact that when the color .660μ was obtained from a spectrum formed by a single prism, slight depressions resulted. The single prism spectrometer with its insufficient dispersion of the red was therefore replaced by one with two prisms as described above.

As the peculiarity of the action of the transition colors is later explained as due to equilibrium between their direct and reflex effects, the transition points will subsequently be termed equilibrium points or colors.

In Fig. 6 the central portions of the above curves, together with that for .657μ, are shown together. The data are collected in Table 2. The reflex effect increases to a maximum for .687μ and thence falls off rapidly with increase of wave length to .740μ which is the longest wave

TABLE 2. *Partial Reflex Curves*

λ	Normal	.657μ	.660μ	.677μ	.687μ	.690μ	.740μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.660μ	0.0174	0.0171	0.0173	0.0174*	0.0172	0.0172	0.0172
.640	.0159*	.0157	.0160*	.0156	.0151	.0156*	.0158
.620	.0152	.0149	.0154	.0148	.0141	.0147	.0148
.590	.0148	.0144	.0147	.0140	.0135	.0139	.0143
.550	.0155	.0150	.0152	.0148	.0142	.0146	.0148
.530	.0165	.0160	.0164	.0161	.0154	.0157	.0159
.500	.0209	.0212	.0207	.0212	.0208*	.0204	.0201

*Values not read, but obtained from full curves.

studied. It seems probable that the reflex effects of still greater wave lengths would quickly become negligible, unless, indeed, there is an

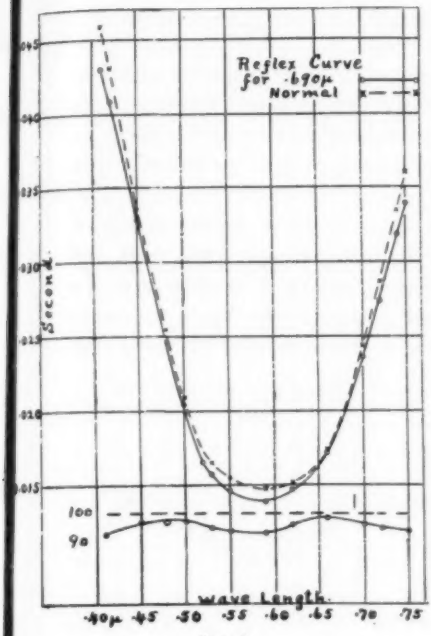


FIG. 2

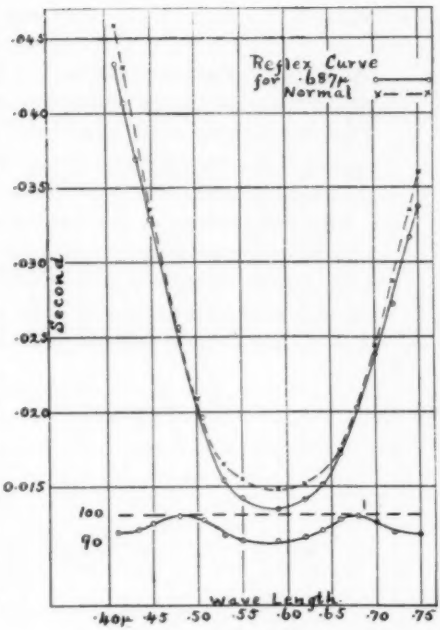


FIG. 3

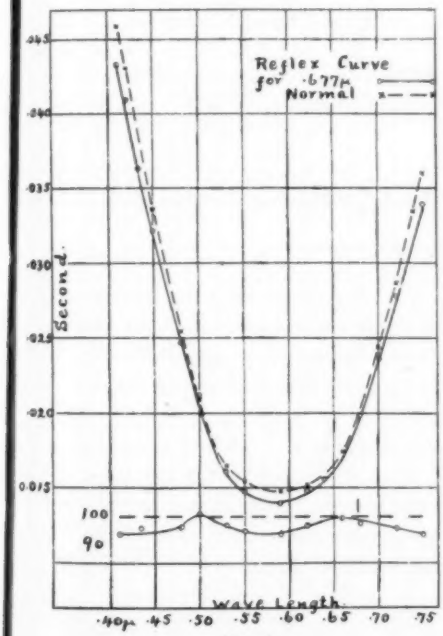


FIG. 4

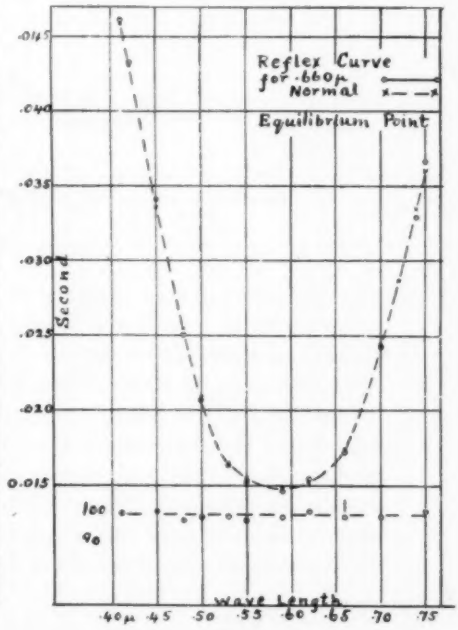


FIG. 5

equilibrium point at some region beyond, a supposition to which no support is given by the sensation curves of Abney and of König, Fig. 34. This diminishing reflex effect with increasing wave length affords an explanation of certain interesting experiments of Edridge-Green, which are later discussed.

The reflex effect of the fatiguing color .625μ is shown in Fig. 7. The three depressions in the curve, corresponding to the ends and middle of the spectrum are well exhibited, and the coincidences as before are at the wave lengths .66μ and .50μ. There can be no doubt that the reds whose reflexes were just studied are simple colors, and

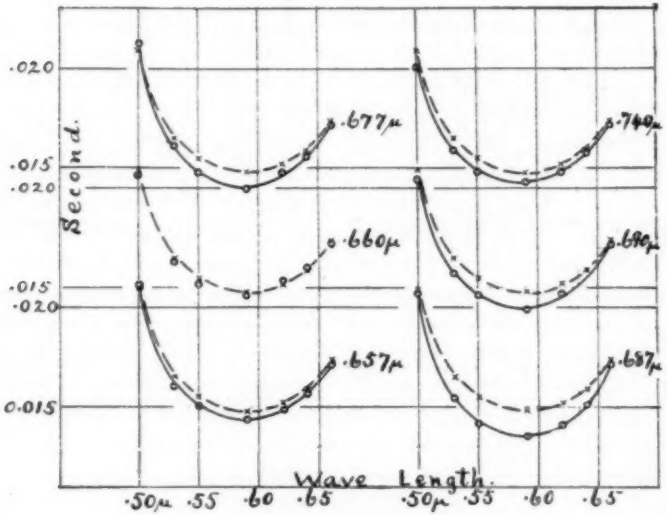


FIG. 6. *Partial Reflex Curves.*

that the color .625μ is compound. Yet the same reflex depressions occur for both. Since red affects all three color sensations that are possible according to the three-component theory, a compound color, .625μ, can affect no more. The inferences to be drawn from the curves in question are therefore not contradictory.

In Fig. 8 the reflex effect of .577μ is displayed. This is near the brightest part of the spectrum, which probably accounts for the large reflex effect obtained. There are two well marked depressions; but the larger, extending from the coincidence point .48μ to the end of the red, has a slight indication of a division into two parts.

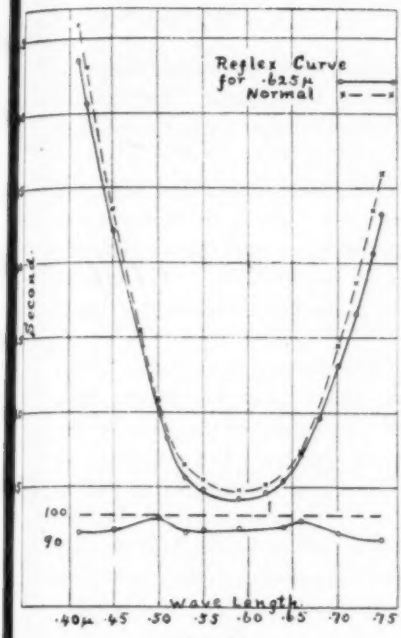


FIG. 7

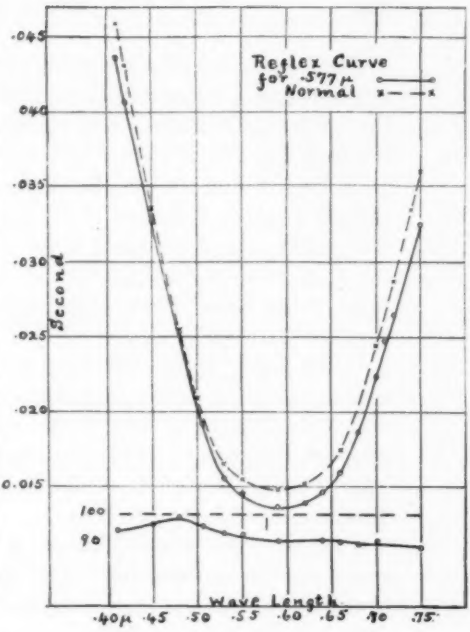


FIG. 8

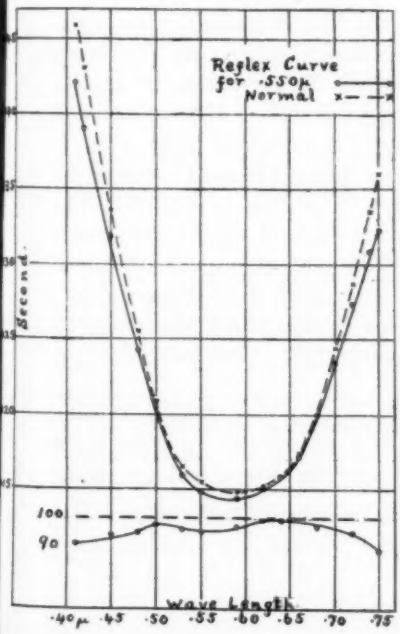


FIG. 9

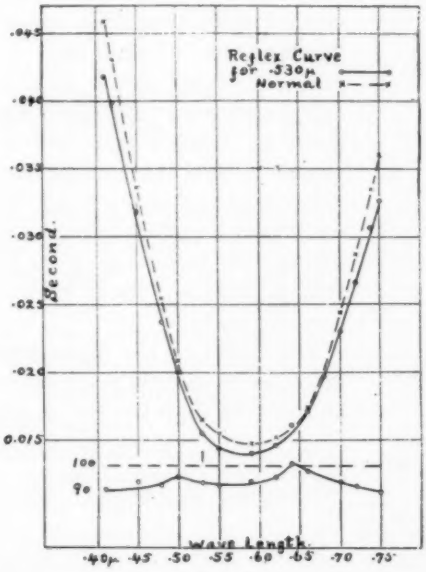


FIG. 10

The second equilibrium color .570μ was then made the fatiguing stimulus and a curve similar to that in Fig. 5 obtained.[8] As with the preceding equilibrium point, complete coincidence with the normal resulted.

In Figs. 9 and 10 are shown the reflex effects obtained with the fatiguing colors .550μ and .530μ respectively. As is to be expected, both curves are of the same type, with the three well marked depressions separated by the coincidence regions .66μ and .50μ. The magnitude of the reflex effects is noticeably greater for the shorter wave than for the longer.

In the paper[9] on the discovery of the transition points, it was found that a transition region extended from about .460μ to .480μ. This was considered to be due to the wide dispersion of the blue in the prismatic spectrum. If that explanation were true, the transition or equilibrium region in the violet should possess a still greater width. This was found not to be the case.

A study of the sensation curves of Abney and of König, Fig. 34, suggested a better explanation. The region lying between .480μ and .520μ is visually by far the most complex in the spectrum, as the sensation curves there have two, and according to some three, intersections, and the curve for red has in that neighborhood also its more refrangible end. It seemed probable therefore that two or possibly three additional equilibrium points might exist close together. As a strongly marked reflex depression always occurs in the green-yellow part of the spectrum, which is also the easiest to measure with accuracy, a series of partial curves for thirteen colors extending from .495μ to .523μ, inclusive, was obtained in a short time. The data for these are given in Table 3 and the curves are all plotted in Fig. 11. The results verified the anticipation. Two additional equilibrium points were discovered at the wave lengths .520μ and .505μ, and the previously known equilibrium point at .480μ was verified. Full persistency curves were now made for each of these colors, which are similar to that in Fig. 5. These curves are completely coincident with the normal.

The effects of the intermediate colors .515μ and .495μ were then investigated with the results shown in Figs. 12 and 13. In both curves are seen the three familiar depressions, and the two coincidence points

[8] As all curves obtained with the equilibrium colors are coincident with the normal, only that in Fig. 5, for .660μ, is shown.

[9] Phil. Mag. *38*, p. 69; 1919.

situated as before. It is especially to be noted that no new parts of the spectrum are enhanced in brightness.

The reflex effects of the two colors .460μ and .450μ are given in Figs. 14 and 15, respectively. The three depressions and the two coincidence points occur as before.

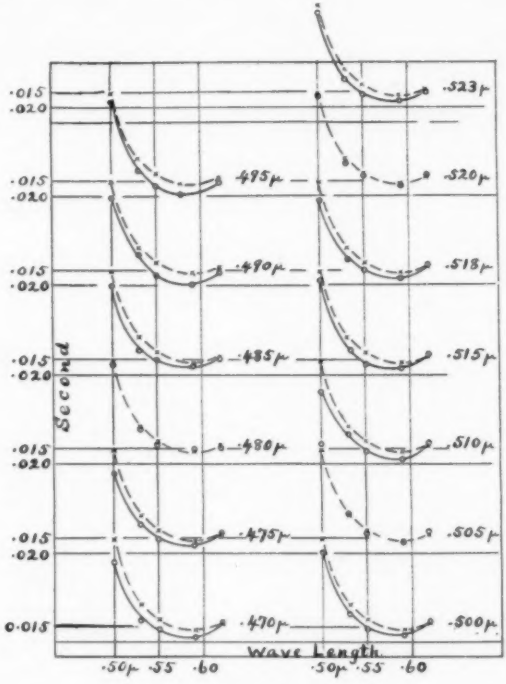


FIG. 11. *Partial Reflex Curves.*

The last equilibrium color as previously determined was of the wave length .420μ. This was now examined for its reflex effect. The curve is shown in Fig. 16. Three small depressions are produced. The color .425μ was then employed with the result that this was proved to be the real equilibrium point. The curve is like that in Fig. 5. As the original points were discovered when darkness adaptation affected fatigue, the discrepancy in the determination of this point, and also in the result for .460μ, seems fully explained.

The last color to be discussed is the violet of wave length .410μ. Some curiosity was aroused as to whether it would show any reflex

TABLE 3. *Partial Reflex Curves*

λ	Normal	.470μ	.475μ	.480μ	.485μ	.490μ	.495μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.620μ	0.0152	0.0152	0.0153	0.0151	0.0151	0.0148	0.0148
.590	.0148	.0143	.0145	.0148	.0146	.0140	.0141
.550	.0155	.0148	.0149	.0153	.0150	.0146	.0146
.530	.0165	.0154	.0159	.0163	.0156	.0160	.0157
.500	.0209	.0193	.0193	.0206	.0199	.0198	.0204

λ	.500μ	.505μ	.510μ	.515μ	.518μ	.520μ	.523μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.620μ	0.0154	0.0153	0.0153	0.0153	0.0152	0.0154	0.0150
.590	.0144	.0148	.0143	.0144	.0144	.0147	.0144
.550	.0149	.0153	.0148	.0147	.0150	.0153	.0148
.530	.0158	.0165	.0159	.0156	.0157	.0161	.0158
.500	.0200	.0213	.0188	.0202	.0196	.0207	.0203

effects at all, on account of its low intensity, which the luminosity measurements of Abney[10] show to be about one seven-hundredth of the brightness of yellow. The result given in Fig. 17 was most surprising. Two very large reflex depressions were obtained, the larger of which covering the more refrangible part of the spectrum from the coincidence point at .66μ is partly divided into two smaller ones at the second coincidence point .50μ.

A comparison of the curves for .410μ and .577μ, Figs. 17 and 8, shows that, so far as luminosity is concerned, violet is quite seven hundred times as powerful as yellow in exciting reflex effects. This is probably explanatory of the conclusions of Rood[11] in respect of complementary colors: "A violet which appears to the eye quite dark, is able to balance a bright greenish-yellow and form with it, white, and the same is true of ultra-marine blue and yellow. There is another way of stating these facts: we can say that in mixtures, violet has a greater power of saturation than any of the colors; next follows ultra-marine blue; then red and cyan-blue about the same, then orange and green about the same, and finally, yellow."

In Fig. 18, the reduced curves at the bottom of each figure including all the equilibrium curves, are gathered together. Each fatiguing color is indicated by a small vertical line below the corresponding curve.

[10] Abney: "Researches in Colour Vision." p. 94.
[11] "Modern Chromatics," p. 165.

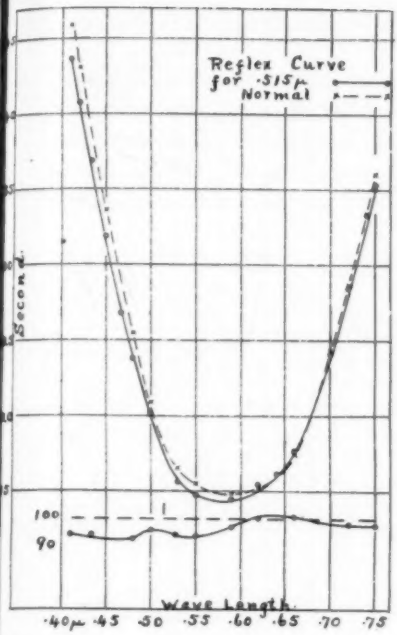


FIG. 12

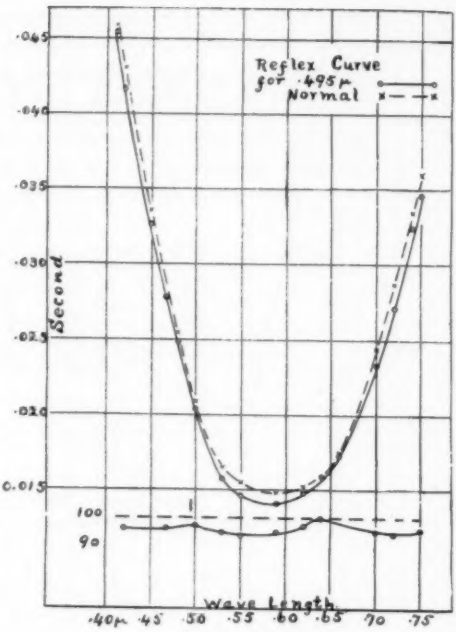


FIG. 13

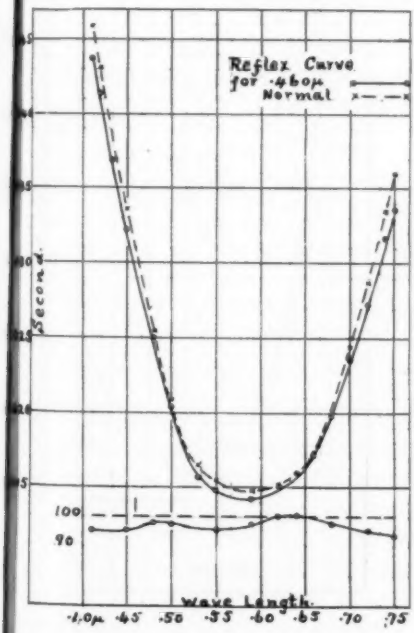


FIG. 14

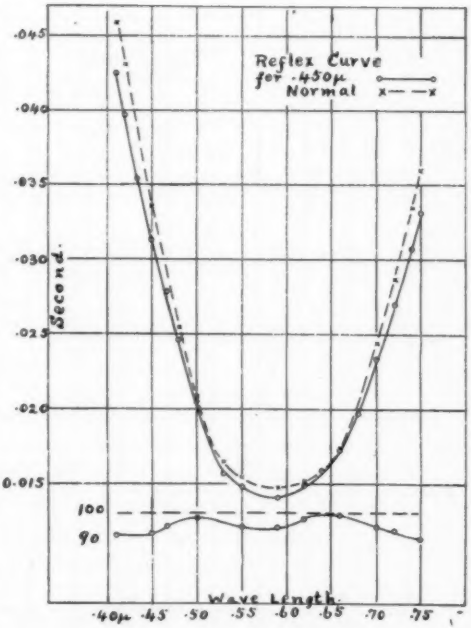


FIG. 15

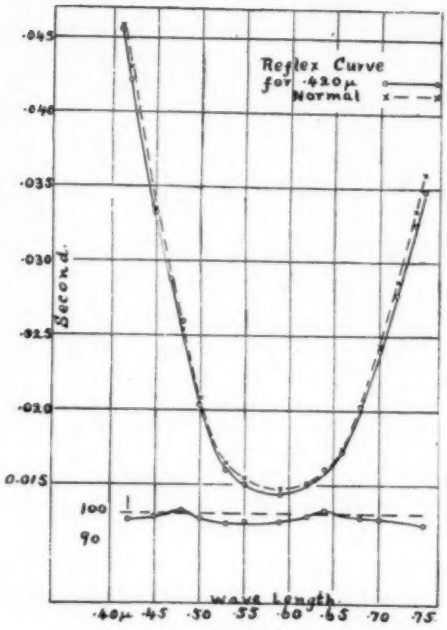


FIG. 16

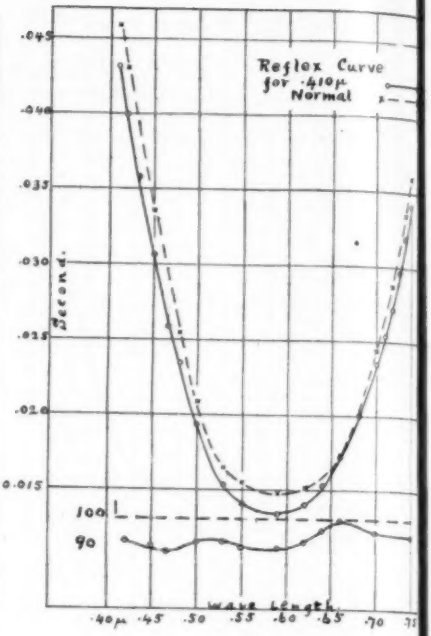


FIG. 17

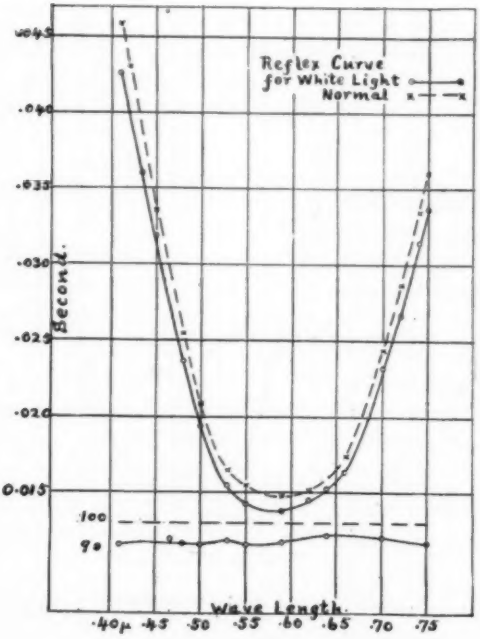


FIG. 19

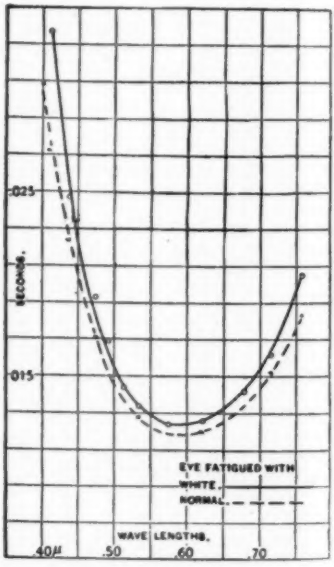


FIG. 20

The depressions and coincidences are all seen together, and the general uniformity in extent and position is evident. The relative depths of the depressions probably indicate the magnitude of the reflex process, but this cannot be fully appreciated unless it can be connected with the luminosity or the wave length of the colors, or with both.

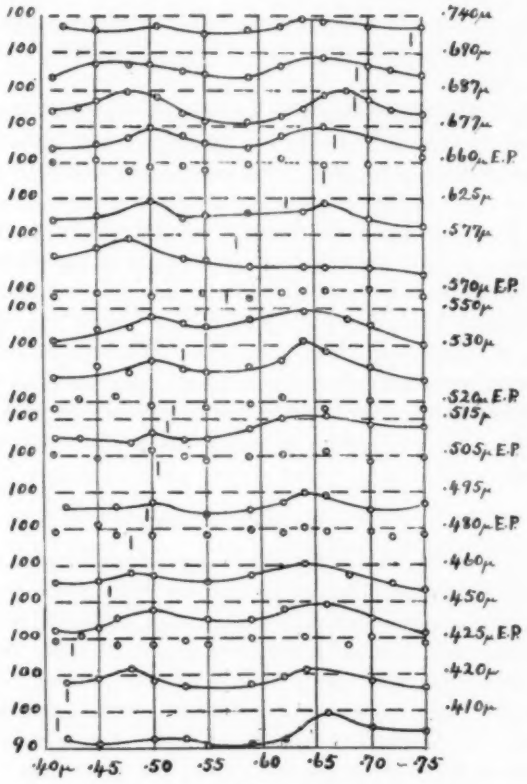


FIG. 18. *Reduced Reflex Curves.*

In these experiments the fatigue was isolated in one eye, and the reflex effects, which occur in both, were isolated in the other, and therefore can be studied by themselves. We are justified in concluding that every ray of light excites in the visual apparatus reciprocal or reflex effects which enhance the brightness of three parts of the spectrum corresponding to red, green, and violet. It is probable that the

relative actions on the three sensations vary with the exciting color, and range from the pronounced effects described in these experiments to the refined gradations accompanying the most delicate shades of color. The magnitude of the reflex sensations seems to be proportional to some extent to the luminosity of the color stimulus, but varies to a far greater extent inversely as some function of the wave length. The strong reflex effects of the violet probably merge gradually into the painful and destructive action which ultra-violet waves exercise on the retina.

If all the reflex curves were to be plotted together, the resultant might be interpreted as the effect to be produced if white light were used for the purpose of fatigue. This experiment was tried with white light and the curve in Fig. 19 plotted from the measurements obtained. The whole curve lies below the normal, indicating that the reflex effect of white light enhances the brightness of all parts of the spectrum. The reduced curve in the lower part of the figure has three slight depressions which show that white light recognizes, as it were, the three color sensations. Possibly this result may have some bearing on the question as to whether white light is a mixture of color waves or an irregular disturbance in the ether which is resolved into periodic elements by the prism. Such evidence as these curves afford inclines to the former view.

III. FATIGUE CURVES

It seemed incredible that reflex effects of such magnitude as have just been described should not betray themselves in the same eye that was fatigued. Yet the numerous curves previously published and referred to above in Section I, were apparently devoid of reflex depressions. While the preceding experiments were in progress, it was recalled that in the author's[12] first paper on color vision very slight depressions had been found and commented upon as possibly indicative of enhancement of sensations. With one exception, the green, they were too small to be of evidential value, and they were gradually forgotten. As the former experiments were made with the unfatigued eye in darkness adaptation, it was finally suspected that darkness itself produced a reflex effect, or something analogous to it, which diminished rather than enhanced the brightness of colors. Certain curves exhibiting the effect of darkness had previously been obtained,[13] here reproduced in

[12] Phys. Rev. *11*, 274; 1900.
[13] Phys. Rev. *11*, p. 265; 1900.

Fig. 33, which showed effects of about the same magnitude as the reflex depressions, but acting in opposition to them. It seemed likely therefore that darkness adaptation of one eye had masked or balanced the reflex action of light in the other.

TABLE 4. *Fatigue Curves*

λ	Normal	.687μ	.670μ	.660μ	.589μ	.570μ	.550μ	.530μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.750μ	0.0360	0.0382	0.0391	0.0358	0.0374	0.0353	0.0310	0.0321
.740	.0335	.0356	.0362	.0338	.0351	.0332	.0293	.0300
.720	.0287	.0299	.0311	.0282	.0297	.0289	.0248	.0264
.700	.0244	.0261	.0261	.0237	.0260	.0245	.0215	.0225
.680	.....	.0205	.0214	.....	.0218	.....	.....	.0192
.660	.0174	.0176	.0181	.0173	.0184	.0171	.0166	.0170
.640	.....	.0160	.0163	.0160	.0160	.0157	.0156	.0158
.620	.0152	.0149	.0148	.0151	.0154	.0150	.0151	.0153
.590	.0148	.0141	.0144	.0147	.0157	.0146	.0150	.0151
.550	.0155	.0147	.0151	.0157	.0175	.0154	.0169	.0166
.530	.0165	.0156	.0159	.0163	.0183	.0167	.0182	.0183
.500	.0209	.0206	.0208	.0207	.0204	.0204	.0202	.0207
.480	.0255	.0254	.0254	.0251	.0241	.0250	.0230	.0233
.450	.0336	.0326	.0329	.0333	.0321	.0328	.0305	.0310
.433	.....	.0372	.0380	.0393	.0365	.0384	.0342	.0362
.420	.0431	.0411	.0414	.0429	.0413	.0427	.0407	.0398
.410	.0459	.0440	.0442	.0465	.0440	.0470	.0436	.0428

λ	.520μ	.515μ	.505μ	.495μ	.480μ	.450μ	.425μ	.410μ
	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
.750μ	0.0362	0.0342	0.0354	0.0341	0.0361	0.0325	0.0362	0.0342
.740	.0335	.0319	.0328	.0321	.0333	.0305	.0337	.0319
.720	.0282	.0273	.0284	.0275	.0286	.0271	.0286	.0278
.700	.0250	.0235	.0243	.0234	.0247	.0236	.0243	.0238
.680	.....	.....	.....	.....	.....	.....	.....	.0194
.660	.0173	.0177	.0174	.0168	.0171	.0181	.0175	.0172
.640	.0162	.0164	.0163	.0155	.0163	.0168	.0162	.0159
.620	.0151	.0157	.0152	.0147	.0154	.0159	.0155	.....
.590	.0148	.0153	.0147	.0142	.0148	.0153	.0149	.0142
.550	.0155	.0160	.0154	.0145	.0154	.0162	.0153	.0146
.530	.0166	.0171	.0162	.0155	.0165	.0168	.0166	.0157
.500	.0202	.0205	.0204	.0200	.0210	.0214	.0205	.0206
.480	.0258	.0241	.0252	.0242	.0252	.0267	.0259	.0270
.467	.....	.0270	.....	.0290	.....	.....	.....	.0304
.450	.0332	.0315	.0335	.0349	.0341	.0351	.0340	.0367
.433	.....	.0365	.....	.0414	.....	.0411	.....	.0420
.420	.0430	.0399	.0428	.0454	.0435	.0466	.0431	.0459
.410	.0460	.0427	.0467	.0488	.0467	.0502	.0456	.0491

A new set of persistency curves was next obtained with the right eye in a state of fatigue and at the same time the left maintained in daylight adaptation. The results completely verified the anticipation, and supplement in a remarkable way those curves already described.

The first color used to fatigue the eye was red of wave length .687μ. The measurements are given in Table 4 with those for all the curves of this series. The results are shown in the curve in Fig. 21. The fatiguing action produced an elevation in the part corresponding to red, and two reflex depressions in the parts corresponding to violet and green. Two coincidence or intersection points are found as before at the same wave lengths .66μ and .50μ. Since red light also produces a reflex enhancement of the red, the elevation is the algebraic sum or the net result of fatigue and reflex action, the balance being in favor of the fatigue. Red light fatigue therefore diminishes the brightness of red, but increases the brightness of both green and violet. In Fig. 22 the curve for the fatiguing color .670μ is shown, which resembles the preceding so closely as to need no comment.

The equilibrium color .660μ resulted in a curve, Fig. 23, which coincides with the normal. This was also the case with the equilibrium color .570μ.[14] The observations made after fatiguing with the intense middle colors of the spectrum were at times rendered troublesome by the after images. In various ways these difficulties were surmounted and consistent readings obtained.

Only one color between the first two equilibrium points seemed necessary; that chosen was .589μ, the curve for which is shown in Fig. 24. This is characterised by two prominent elevations in the red and green, and a reflex depression in the violet. The coincidence points are at nearly the same places as before. Since the charactertistic effect of fatigue is a diminution of brightness, elevations will always indicate what sensations are directly, and depressions what are reflexly affected. Yellow light therefore directly stimulates the red and green sensations, and by the sensory reflex process, the violet.

Two colors in the green were used, .550μ and .530μ, the curves for which are shown in Figs. 25 and 26. The effects produced are identical, consisting of an elevation in the green and depressions in the red and violet. The points of coincidence or intersection are about the same as before.

[14] As all the equilibrium colors give curves coincident with the normal, only that for .660μ is here reproduced.

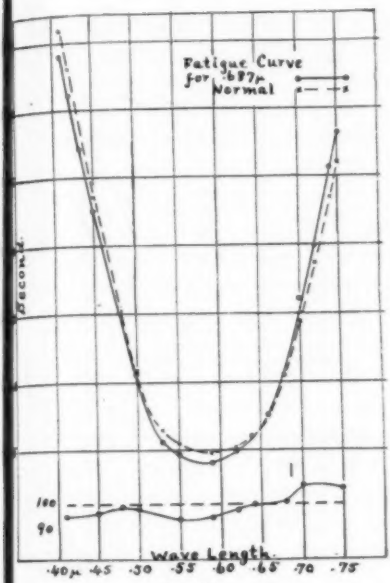


FIG. 21

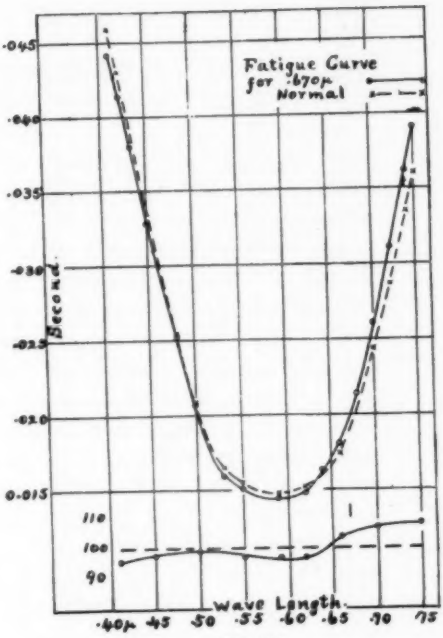


FIG. 22

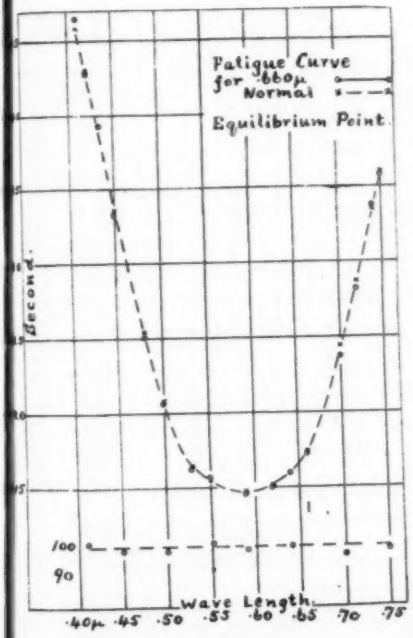


FIG. 23

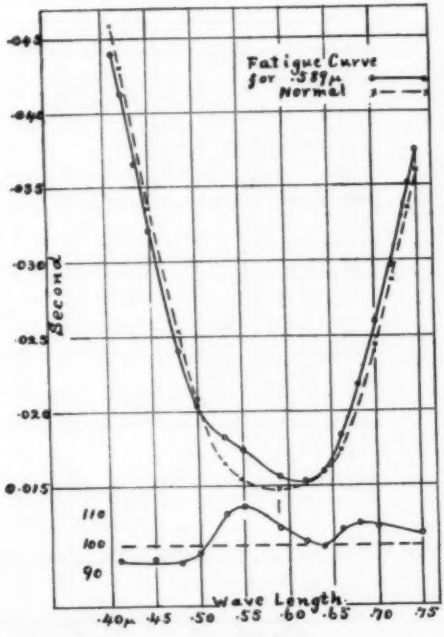


FIG. 24

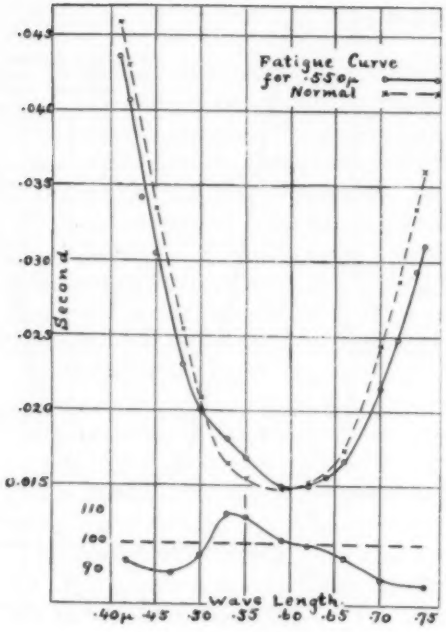


FIG. 25

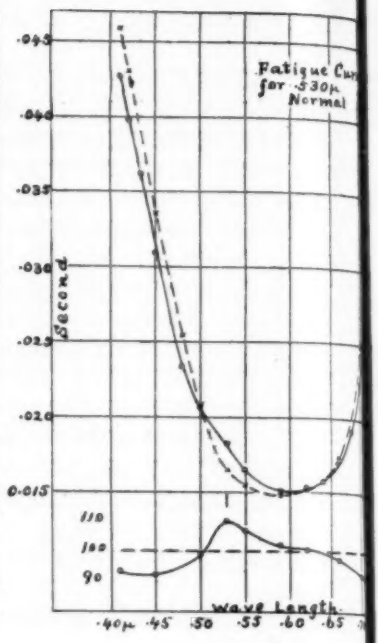


FIG. 26

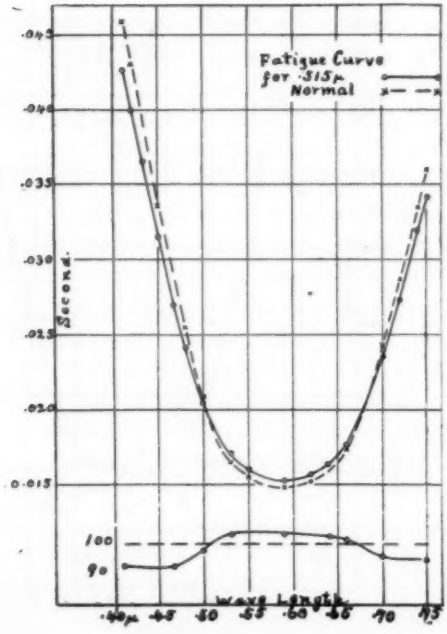


FIG. 27

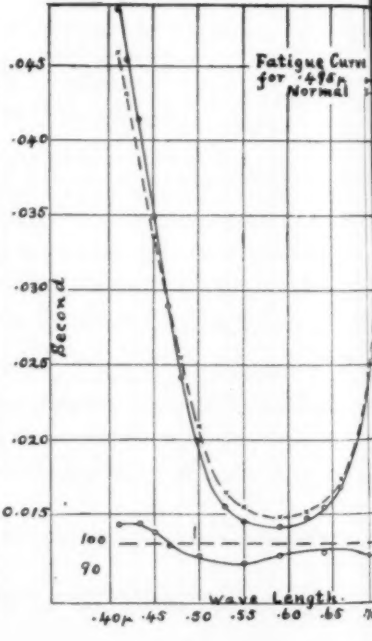


FIG. 28

The equilibrium colors previously found at .520μ, .505μ, and .480μ, proved to have the same character when used directly. The corresponding curves are similar to that in Fig. 23.

The curves for the fatiguing colors .515μ and .495μ, between these equilibrium points, are plotted in Figs. 27 and 28. They are especially to be noted. All curves from the red to .515μ, inclusive, have elevations only in the red or green, or in both. The violet is affected always so as to produce a reflex depression. Beginning with .495μ, the remainder of the curves show an elevation in the violet, indicating this color sensation to be directly stimulated. The curve for .495μ is almost identical with that for violet of wave length .410μ, as shown in Fig. 30. Both have depressions in the green and red portions of the curve. The direct effect of the color .515μ is evidently in the green, while on the more refrangible side of the equilibrium point .505μ, violet is the sensation directly excited. An examination of this region in the sensation curves of Abney and of König, Fig. 34, indicates that, at the intersection of the green and blue (or violet) sensation curves, a very small change of wave length transfers the predominating stimulus from green to violet. It will be seen later that the equilibrium color .505μ is probably identical with this intersection point. It follows therefore that these persistency curves are in close harmony with the sensation curves.

From Fig. 29 it is seen that the effect of the fatiguing color .450μ is to cause elevations in the violet and green regions and a depression in the red. This indicates that by this color the two sensations, green and violet, are both directly stimulated, while the red is enhanced by the reflex process. The evidence from Fig. 28 that the color .495μ, which lies between the green and .450μ, does not directly affect the green sensation, but only reflexly, is a most unexpected result for which the author has no explanation. It is quite possible from the sensation curves of Abney and of König, that for a short distance in this region of the spectrum the red and green sensations nearly balance each other.[15] In this characteristic of the sensation curves the explanation may lie. Notwithstanding the somewhat minute analysis of this part of the spectrum described in this paper, further investigation of this peculiarity is necessary.

The effect of the remaining equilibrium color .425μ shows coincidence with the normal throughout, as in Fig. 23.

[15] See especially Exner's Sensation Curves in Parsons', Colour Vision, p. 223.

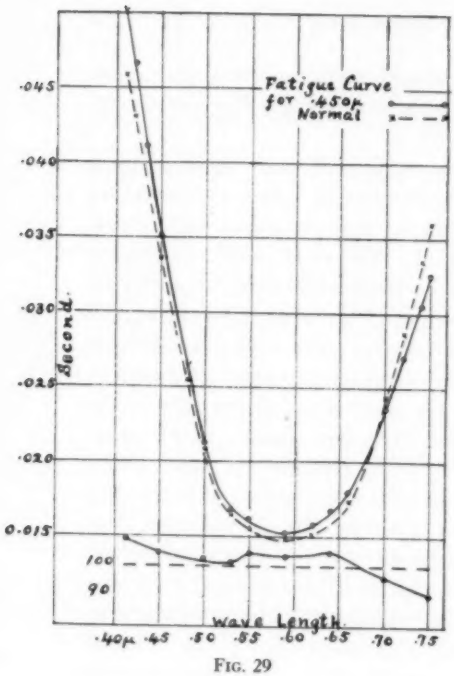


FIG. 29

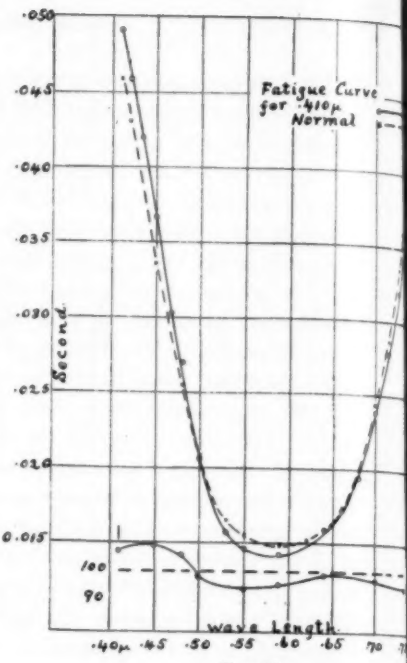


FIG. 30

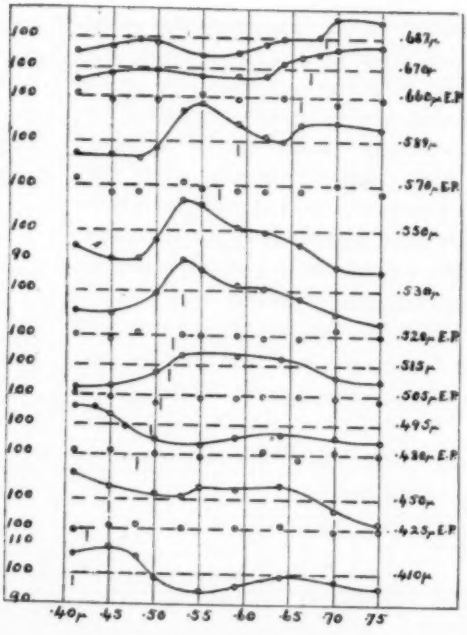


FIG. 31. *Reduced Fatigue Curves.*

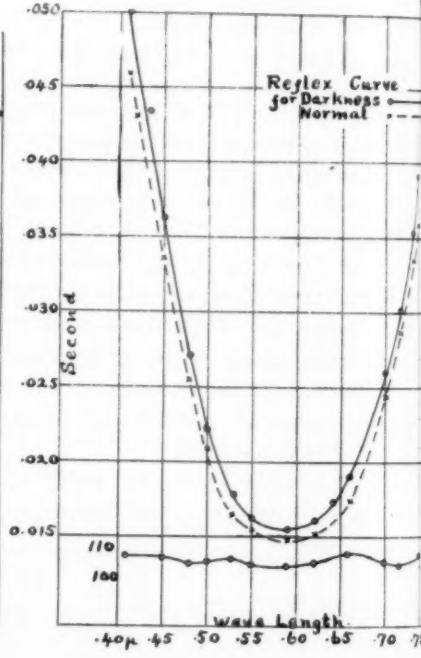


FIG. 32

The reduced fatigue curves are all shown together in Fig. 31, which displays comprehensively the normal curves, the elevations and depressions of the fatigue curves, the intersection points, and the fatiguing colors indicated by short vertical lines below each normal.

From a former paper[16] there is reproduced in Fig. 20, the persistency curve obtained when the observing eye was fatigued with white light. This curve lies above the normal in all parts indicating fatigue of all the sensations. No indication of a division into three elevations is found. If this curve be compared with the reflex curve for white in Fig. 19, the opposite character of the two actions will at once be evident.

The above curves show that the red, green, and violet colors produce one elevation in the corresponding part of the spectrum, and two depressions in the parts of the spectrum corresponding to the other two sensations. The yellow and blue colors produce two elevations, the former in the red and green regions, the latter in the green and violet, and a single depression in the part corresponding to the remaining sensation. No color was found that produced elevations simultaneously in the red and violet. Abney[17] came to the conclusion that the sensation of violet was compounded from the red and blue sensations. The evidence presented by both sets of curves in this paper is completely against that view, and it seems impossible to avoid the conclusion that violet is a simple sensation. The compound character of yellow and blue is abundantly confirmed.

It was stated above that the effect of darkness adaptation of the left eye might be transferred to the right (or *vice versa*) and so balance and thereby conceal the reflex effect of whatever color stimulus was applied. In order to prove this, a curve was obtained by the right eye in normal daylight adaptation, the left eye being kept blindfolded for about three hours, while the measurements were made. The data are given in Table I and the curve is shown in Fig. 32. This lies wholly above the normal, showing that the effect of darkness adaptation is transferred from one eye to the other, and diminishes the brightness of all color sensations, an effect just the opposite of that shown for white light in Fig. 19. The reduced darkness curve in Fig. 32 shows evidence of three depressions, and thereby adds its testimony to the threefold character of color vision.

From a former paper[18] there is reproduced in Fig. 33, a set of curves obtained under different conditions of darkness adaptation. In this

[16] Phys. Rev. *11*, p. 276; 1900.
[17] Researches in Colour Vision, p. 230. Also: Phil. Trans. A. *193*, p. 259.
[18] Phys. Rev. *11*, p. 265; 1900.

figure the curves marked "a" and "b" were obtained by the right eye, when both were under the influence of very dim light or darkness. The curve "c" was obtained when the right eye was blindfolded and the left was in daylight adaptation. The curve "d" is the normal.

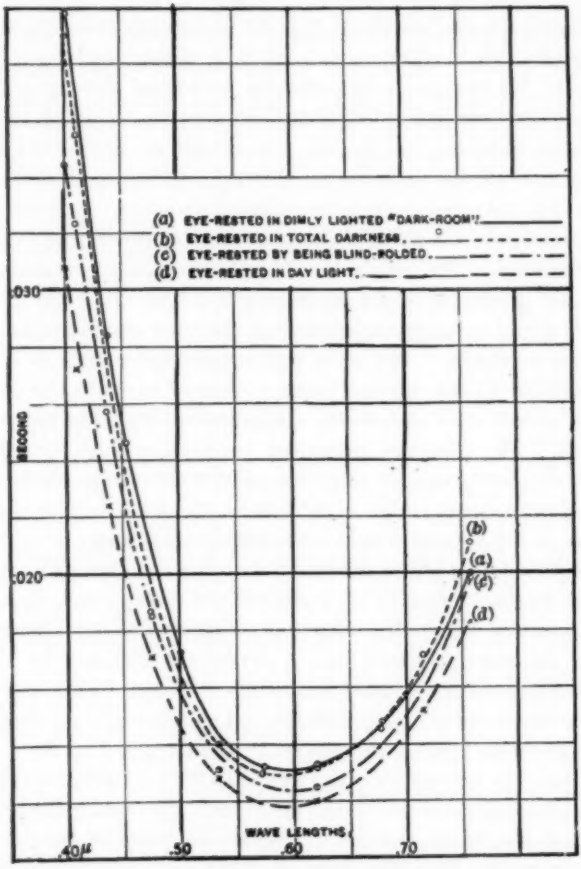


FIG. 33. *Curves showing various degrees of darkness adaptation.*

Under all conditions therefore the effect of darkness adaptation is precisely the same in character, though not in magnitude.

Since darkness adaptation always diminishes the brightness of all colors, it may be questioned whether darkness makes the eye more sensitive, as is usually stated. From these experiments it seems rea-

sonable to hold that the effect of prolonged darkness adaptation is to give time for all direct and reflex effects of light—some of which, according to Burch,[19] last for quite two hours—to disappear from the visual apparatus. Very feeble rays of light will therefore become perceptible as there is nothing present in the retina to interfere with them. It must be remembered, however, that the curves described in this paper are for bright colours which involve the photopic mechanism of the eye, and the results may not be valid for the scotopic mechanism, if such exists.

The experiments on darkness are susceptible of another and probably a more rational explanation, which does not attribute to darkness a positive action. We have seen that the effect of white light fatigue on one eye is just the opposite of its reflex effect from the other. The normal condition of the retina, as experiment shows, is obtained when daylight is admitted to both eyes simultaneously. But daylight must exercise some effect, both of fatigue and reflex action. By blind-folding one eye, the white light is thereby excluded and prevented from having its reciprocal or reflex effect. The apparent reflex effect of darkness is therefore merely due to protection from the real effect of white light.

From the results of these measurements, it is evident how extraordinarily far from correct is Charpentier's[20] conclusion that darkness adaptation of one eye has no effect on the other, reliance upon which has probably invalidated many measurements of color.

On reference to the darkness adaptation curves in Fig. 33, it will be observed that the maximum effect in diminishing the physiological brightness of the spectrum is not obtained by blind-folding the eye or by keeping it in complete darkness, as would quite naturally be expected, but by adapting the eyes for light of very low intensity, such as that in a dimly lighted dark room. The dark room used in this experiment was painted black and the only illumination was the light escaping from the screens around the acetylene flame. The curve "c" for the blind-folded eye is about half way between the normal "d," and the curve "a" for the eye adapted for weak light.

This unexpected peculiarity serves to explain an observation by Fechner which is thus described by Edridge-Green[21]; "In certain conditions the additional stimulation of one eye with light not only

[19] Proc. Roy. Soc. B, *76;* 1905.
[20] Parsons: Colour Vision, p. 50.
[21] Physiological Optics, p. 127.

causes no sensation of increased brightness but the reverse. This is illustrated by Fechner's paradoxical experiment. One eye is directed to an illuminated surface, as for instance a sheet of white paper, whilst the other eye is closed. If the closed eye be suddenly opened behind a sheet of smoked glass, the field of vision will appear distinctly less bright."

From the curves it is seen that blind-folding or closing one eye will diminish the brightness of the white surface to the other. When weak light coming through the smoked glass is then admitted to the previously closed eye, a still further diminution of the brightness of the white light ought to result, which is what Fechner observed. Fechner's experiment therefore follows logically from the evidence of the curves, and ceases to be paradoxical when the comparative effects of weak light and darkness upon the visual apparatus are more perfectly understood.

In Fig. 34, the sensation curves[22] of König and of Abney are shown with the six equilibrium points marked. The occurrence of these points at the ends and intersections of the sensation curves is of great interest. The actual wave lengths of the intersections for different observers are given by Parsons[23] in the following table, to which four of the equilibrium points are added by the author:

	a	*b*	*c*	*d*
König (sunlight)	573	503	496	450
Exner (arc light)	577	508	494	475
Abney (arc light)	577.2	515	500	...
Equilibrium points (arc light)	570	520	505	480

The equilibrium point .480μ is probably not the intersection point "d," the existence of which Abney's curves do not confirm, but rather the end of the sensation curve for red. The agreement of the author's values with those of Abney is quite good.

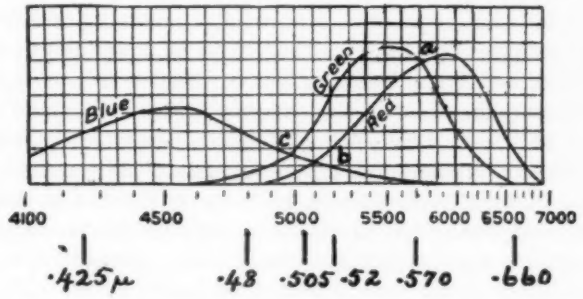
Since the six equilibrium points apparently produce neither direct nor reflex effects, we may interpret this as an equilibrium between the two actions, or possibly as a balance between excitation and inhibition.

The two mean points of coincidence with the normal which nearly all the curves exhibit, are, for the fatigue curves, .648μ and .50μ, and for the reflex curves, .658μ and .502μ. The further mean value .653μ

[22] Parsons: "Colour Vision," pp. 222 and 244. See also Exner's curves on p. 222 for intersection point "d."

[23] Parsons: loc. cit. p. 230.

is near the equilibrium point .660μ, and may be identified with it. The second is almost the mean of the three equilibrium points .480μ, .505μ and .520μ.



Normal Sensation Curves. (Abney and Watson.)

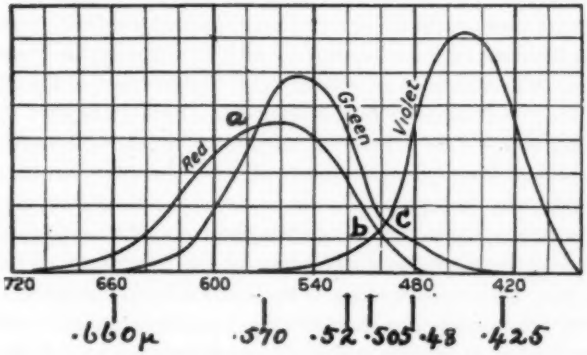


FIG. 34. *Sensation Curves. (König and Dieterici.)*

IV. THEORETICAL APPLICATIONS

From the experimental results which have now been considered, we learn that the action of light of every hue upon the retina is not unitary but dual in its nature, one part consisting of a direct effect and the other of a reflex. The direct action affects the one sensation involved if the color is red, green, or violet, or the two sensations if orange, yellow, or blue. The reflex process affects all three sensations in every case. According to the three component theory of color vision it is generally held[24] that equal and simultaneous stimulation of the three

[24] Abney: Researches. p. 231.

primary sensations results in the perception of white. But even if the several stimulations need not be equal, no doubt a definite ratio of excitations is involved in the white process.

When, for example, red light falls upon the retina, the sensatory or sensory reflex excites the three fundamental sensations sufficiently to cause the perception of white; the direct action of red affects only the corresponding sensation, resulting in the simultaneous perception of red. What appears finally to consciousness therefore is the red color overlying, as it were, a substratum of white of an intensity depending on the strength of the original red stimulus. The same reasoning applies to all color impressions, except that in the case of compound colors, two sensations are directly involved. If the color excitations are very intense, the sensation of white may be so powerful as to render the direct effect of the color almost vanishingly small. Hence all colors when of great intensity would approximate whiteness in appearance.

While this view seems to be completely in harmony with the experimental evidence, yet color perception may be expressed in other terms which offer some advantages. Every color affects part of the receptors directly and the remainder in the same retinal area reflexly. These in every case, include the whole of the colors of the spectrum which when combined become white light. Hence the two portions of the color process, the direct and the reflex, occupy qualitatively a complementary relationship. When any sensation is affected directly and fatigued, with a consequent diminution of the brightness of the color stimulus, the luminous response of the remainder is enhanced. These two effects are not usually quantitively capable of exactly balancing each other and so producing white. At moderate intensities the whole of the reflex effect will unite with some proportion of the direct and produce white, the residuum being the color. From these considerations color perception may be brought under Sherrington's Principle of Reciprocal Innervation, if that Principle may be enlarged to include sensations within its scope.

The white light underlying all color perception has been widely recognized. In Hering's theory,[25] a separate visual process is assigned to the white-black sensation. In Wundt's theory, it is assumed that "two different stimulation processes are set in action by every retinal

[25] For valuable expositions of the chief modern theories see "Colour Vision," by J. H. Parsons.

excitation, a chromatic and an achromatic. Both stimulations follow different laws with increasing stimulus intensities. At low intensities the achromatic surpasses the chromatic; at moderate intensities the chromatic is the relatively stronger; at the highest intensities the achromatic regains superiority."

Mrs. Ladd-Franklin assumes that "in the earliest stages of its development the visual sense consisted of grey." Out of this achromatic sensation the color sensations developed by a process of molecular differentiation.

McDougall provides an independent white process in addition to the three fundamental sensations, red, green and violet. From his numerous researches, Abney[26] concluded that "it appears as if light were the fundamental sensation caused by the main vibration generally, whilst color is, as it were, an overtone to which the receiving nerves are less susceptible than to light, the further away they are situated from the centre, and may be due to the form of vibration."

In some of his later work Abney[27] also inclined to the view that he had possibly separated the white from the color sensations. But according to the experimental evidence adduced in this paper, such a separation is impossible with the direct action of light.

Parsons[28] also states: "But we are confronted with another fact, less easy of explanation, viz., that great increase of intensity of the light not only alters the hue, but also alters the saturation, so that eventually it produces only the sensation of white light. It would seem therefore that luminosity is in some recondite sense an inherent "whiteness" in the color itself, differing in degree in different spectral colors, and varying with the intensity of those colors. Clearly we are here, at the outset, face to face with a physiological fact of immense importance, and much of the difficulty of color vision is concerned with this fact."

But not alone are intense color sensations achromatic. The same phenomenon reappears when the intensity of colors falls very low. So important has this peculiarity been deemed that a special theory for its explanation has been formulated by von Kries,[29] called the Duplicity Theory, which has received wide acceptance. This theory recognizes the existence of two kinds of vision, the photopic, which is chromatic,

[26] Phil. Trans. A., *190*, p. 195; 1897.
[27] Phil. Trans. A., *205*, p. 348.
[28] "Colour Vision"; p. 29.
[29] Parsons: "Colour Vision," p. 203.

and the scotopic, or vision at very low intensities, which is achromatic. Two different mechanisms, the cones and rods respectively, are assumed to be involved in producing these very different types of vision.

But is such a theory necessary? All effects concerning chromatic and achromatic vision seem susceptible of explanation by assuming that at high intensities all colors excite balancing reflexes of equal magnitude which combine to produce white. As the intensities of the colors become smaller, the intensities of the reflexes also diminish, but at a more rapid rate, leaving a greater or less unbalanced residuum of color, which is that normally perceived. Finally, as the intensities of colors are further reduced the reflex sensations again become equal in magnitude with resulting achromatic vision. The whiteness which is always present appears to consciousness usually as a modification of the saturation of color. At medium intensities, color predominates over whiteness; at extremes, both high and low, whiteness equals or predominates over color. Other peculiarities of vision such as night-blindness, may render the duplicity theory desirable or even necessary; but it seems quite superfluous in connection with the very phenomena which it was chiefly designed to explain.

There is universal recognition that spectral colors are far from being saturated. This has found expression by color theorists generally; and it has been specifically stated by Helmholtz, Hering, König, and Exner, for example, that the colors they selected as primary were all to be regarded as more saturated than their spectral values.

Reasons have been given why a color cannot be seen directly apart from its underlying white. Saturation is at once desired and impossible to secure. Increasing or decreasing the intensity of stimulation too far results in lessening the saturation still further. While direct action apart from reflex cannot in its very nature be obtained, the reflex may exist apart from the direct in the form of after images, when the direct action of light has ceased. In this connection, certain observations by McDougall are of remarkable interest and importance, and are in full agreement with the principle of reflex color sensations. According to MacDougall,[30] "An important feature of the after-images of bright white light is that, after a first short period in which two colors fuse to give yellow, or, as is the case after the brightest lights, all three fuse to give white, the colors that in turn occupy the area of the after image, alone and unchanging for considerable periods,

[30] Quoted from McDougall by Parsons: Color Vision, p. 111.

are red, green, and blue only, and these are in every case of exactly the same color-tone, although varying in brightness in different cases and in different stages of one after-image. The red is a rich, crimson red, decidedly less orange than the red of the solar spectrum, the blue is a rich ultramarine, and the green a pure green, having no inclination towards blue or yellow..... They are the purest, richest, most saturated colors that I have ever experienced, and I believe that in this way, and this way only, one may experience absolutely simple, i.e. unmixed and fully saturated color sensations."

It would scarcely be possible to imagine experimental evidence more in harmony with the principle of reflex color sensations than these observations of McDougall.

There seems no reason to doubt that simultaneous contrast can be brought under the operation of this principle. "For the most important element in color contrast is the opponent or complementary effect,"[21] which is precisely what the principle of reflex sensations would lead one to expect. The fact that the phenomenon is reciprocal as well as complementary, is also in harmony with this principle. Indeed, binocular contrast is explained by the curves presented above. For example, if red light falls on the right eye and green on the left, two effects are produced on each retina. The red light causes fatigue of the red and enhancement of the green and violet on the right retina. The effect of green on the left retina is transferred to the right in the form of a reflex enhancement of all three sensations. Some modification of the appearance of the red is bound to occur, which is connected with its complementary. Similarly in the left eye, the appearance of the green color is modified by the reflex influence of red. The combined reflex and fatigue curves therefore prove that contrast effects must occur, and prove also that their mutual influences are in complementary directions.

Since the equilibrium colors do not change the normal curves, it seems probable that pairs of these colors should not produce any contrast effects, either binocular or monocular. The equilibrium colors .660μ and .570μ, for example, should not modify each other's appearance in the least. Indeed, this method might be used to verify the equilibrium points.

The familiar experiment on successive contrast also has new light thrown upon it by the principle of reflex stimulation of the color

[21] Parsons; loc. cit., p. 128.

sensations. If a patch of red color is intently observed for twenty or thirty seconds and then a white surface is looked at, a patch of green of the same shape as the original red is seen. The explanation usually given is that the red receptors are fatigued by the red light, while those for the other sensations are either less or not at all affected. Consequently the white light excites the green and violet receptors much more than the red.

This explanation must now be modified by remembering that while the red sensation is being fatigued, the remainder are being reflexly enhanced. This great enhancement of the green sensation and to a less extent of the violet, is responsible for part and probably the major part, of the observed complementary effect.

The explanation of complementary colors is very direct. Two colors are complementary when their combined direct and reflex effects result in equal stimulation of the three fundamental sensations, or in such proportionate stimulations as fuse together to produce white.

In his consideration of the Young-Helmholtz theory, Parsons[32] has briefly summarized some criticisms as follows:—"The greatest difficulty, however, is experienced in explaining the facts of induction. In general terms it is not difficult to conceive a diminution in response of the components in one direction, associated with an increase in another, after previous stimulation. We might thus account for the increased response to the complementary after stimulation with a given light. Indeed, such a view falls in well with other physiological findings, so admirably elaborated in Sherrington's work. If all spectral lights act upon all three components, then the increased response to the complementary after previous stimulation with a color can be explained, and this was the view adopted by von Helmholtz. It lands us, however, on the horns of a dilemma, for the facts of dichromatic vision —and the same is true of trichromatic—show that lights of greater wave length than about .550μ do not act at all upon the violet component, since no standard blue has to be mixed with the standard red to match colors in this part of the spectrum. Yet the saturation of yellow (.589μ) is undoubtedly increased by previous stimulation with the complementary blue. Von Kries has shown that this effect is not to be explained by any alteration of the intrinsic light of the eye, but is caused by a quantitative change in the response to the secondary light. If yellow light acts only on the red and green compo-

[32] loc. cit. p. 218.

nents, we cannot explain on the theory the increase in saturation which follows previous activity in the blue components. Hess refuses to admit that the facts can be brought into line with the theory. We must therefore accept the theory as explaining satisfactorily *either* the phenomena of after-images *or* those of dichromatic vision, but not both. The large mass of accurate evidence which has now accumulated, chiefly from the observations of Abney and von Kries and their fellow-workers, showing that all spectral lights do not act upon all the components makes it impossible to accept the explanation as it stands for successive induction. Moreover, successive induction is a complex condition allied to simultaneous induction, and it must be admitted that the facts of simultaneous contrast cannot be explained directly by the trichromatic theory alone."

The experimental results presented in this paper give a very exact and comprehensive answer to these criticisms. It has been found by experiment and abundantly shown in the above curves, that a diminution of response of the components in one direction is always associated with an increase in another, unless interfered with by darkness adaptation, and in a perfectly systematic manner. The assumption of Helmholtz that all spectral lights act upon all three components is found to be experimentally verified, though the connection is not by the direct action, as doubtless was Helmholtz's view, but by the reflex influence of light. It is found also that light of every wave-length, including that between .550μ and .740μ has an undoubted effect on the violet. The general failure to detect this is probably due to the neutralizing action of darkness adaptation.

Yellow light acts not only on the red and green components, but as the curves show, on the violet component as well. Attention has been directed also to the great reflex effect on the green-yellow which results from the action of the violet, as well as blue. This accounts adequately for the increase in the saturation of yellow by previous stimulation with the complementary blue.

The accuracy of the observations of the investigators mentioned in the above quotation need not be questioned. But their conclusion that all spectral lights do not act upon all components is conclusively negatived by the present experimental results. Possibly the method used by the author possesses somewhat greater delicacy than theirs, or more probably the transferable effects of darkness adaptation from one eye to the other may have concealed these phenomena from them just as it did from the author for some years.

Finally, in a manner similar to the explanation of binocular contrast, it is quite probable that the facts of simultaneous induction or contrast can be explained by the trichromatic theory when enlarged by the inclusion of the principle of reflex visual sensations.

The results of the experiments in this investigation are strongly opposed to the conclusions of Burch,[33] concurred in by Edridge-Green,[34] to the effect that "fatigue of the eye for any one color does not increase its sensitiveness for any other color." Edridge-Green further says that "if the eye be fatigued for yellow the blue of the spectrum is considerably diminished, not increased," and suggests that the effect is probably due to luminosity contrast.

If, however, the experiments of the latter were performed in a dark room, the large effect of darkness adaptation of the non-observing eye on the other will considerably overbalance the enhancement of the blue due to the reflex effect of the yellow. Since the coincidence point of the normal and other curves occurs at the junction of green and blue in the spectrum, no enhancement of this region takes place and therefore the effect of darkness adaptation is unopposed. Whatever hue of blue is selected, the observation of Edridge-Green is explained without introducing any other principle.

The reflex effects of color are of such magnitude in enhancing the visual sensations that it seems probable some method of seeing the reflex colors could be devised. While considering this question during the progress of the investigation it occurred to the author that some experiments described by Bidwell might be what were sought for.

The result of the sensory visual reflexes is, as we have seen, to reduce the saturation of colors by the production of white. But the impact of light on the retina arouses the reflex only after an interval of time which must be very short, as it is rarely noticeable. It follows therefore that colors falling on the retina should first be perceived for a very short interval of time much more saturated and brighter than in normal vision. Now Bidwell[35] states that "immediately upon the impact of the light there is experienced a sensation of luminosity, the intensity of which increases for about one-sixtieth of a second; much more rapidly towards the end of that period than at first. Then ensues a sudden reaction lasting also for about one-sixtieth of a second in virtue of which, the retina becomes partially insensible to renewed or continuous

[33] Phil. Trans. B. p. 5; 1899.
[34] Physiology of Vision, p. 235.
[35] Proc. Roy. Soc., *56*, p. 132; 1894.

luminous impressions. The stage of fluctuation is succeeded by a sensation of steady luminosity, the intensity of which is, however, considerably below the mean of that experienced during the first one-sixtieth of a second."

These observations are quite in harmony with the principle of reflex sensations.

Further, Bidwell[36] describes an experiment as follows: From a disk a sector of 45° was removed. One half of the remainder was covered with black velvet and the other with unglazed grey or buff paper. A red or other colored card is placed back of the disk in such a position that it can be seen through the open sector. The disk and card are well illuminated with white light. When the disk rotates from 6 to 8 times per second the red ceases to be visible, and in its place is seen the complementary color.

The explanation given by Bidwell and concurred in by Abney[37] is that the brief flash of red light fatigues the red receptors which thereby respond much more feebly than the green sensation when the white light reflected from that part of the disk in its turn reaches the eye; or that the effect is due to successive contrast after an exceedingly brief stimulus.

From the standpoint of this paper, the explanation is altogether different. The initial impulse of direct light, much brighter than normal, arouses the reflex sensations in the brief interval of its duration, from 0.02 to 0.015 second, and is then cut off, allowing the complementary portion of the reflex sensations, which ordinarily is overpowered by the continuous red light, to become visible. Since the eye must always be under some influence, whether of light or darkness, the continuance of either of which alone is fatal to the visibility of the effect, the retina must be maintained in a state of sensitive equilibrium by applying the two opposing effects, light and darkness, alternately. For, according to Bidwell, the brief application of light enhances the brightness, and we have shown that darkness always diminishes the brightness of colors. This explanation does not involve fatigue in the least. Negative after images are the various reflexes dying out in unequal times; the Bidwell effect is the reflex sensation in its beginning.

If this explanation of the phenomenon is correct, we can bring under the principle of reflex sensations, the hitherto curious and fleeting

[36] Proc. Roy. Soc., *61*, p. 269; 1896.
[37] "Researches in Colour Vision." p. 396.

color effects and recurrent images in the other disk phenomena of Bidwell, as well as in those of Sherrington, Benham and others.

Edridge-Green[38] has recently published some observations which may with advantage receive brief consideration. He formed a white with a mixture of certain red, green, and violet colors. After fatigue with red of wave length .670μ the white light was viewed and found to be green in appearance. By reducing the green component to half its original amount, whiteness was restored. No change in the original white was seen when the eye was fatigued with red of wave length, .780μ. This effect is interpreted by Edridge-Green as opposed to the three-component theory of color vision. On referring to the reflex curves for red, Fig. 6, it will be found that the color .670μ produces a fairly large reflex in the green. Fatiguing with this color therefore not only diminishes the intensity of the red but enhances the brightness of the green to such an extent that a smaller portion is sufficient to reestablish the balance and so restore whiteness. The reflex obtained for .740μ is very small, and as the figure shows, the longer wavelengths show diminishing reflexes. Therefore the reflex enhancement of the green from fatigue with .780μ will scarcely be noticed. No appreciable disturbance of the white equation will probably result.

Edridge-Green further states that when the eye is fatigued with the color .760μ and the red .670μ is viewed, it appears yellow or greenish-yellow, which, he states, could not be the case if the red sensation only had been affected.

In explanation of this effect, it may again be stated that the color .740μ produces a distinct reflex in the green-yellow region, and it is probable that .760μ will also have an appreciable effect. As no doubt all colors produce reflex effects of varying magnitude, the change of color observed is explained.

From Fig. 6 it seems probable that the Edridge-Green white equation will not be affected by fatiguing the eye with the equilibrium color .660μ, and that the disturbance of the balance of colors will increase to a maximum with a fatiguing stimulus of .687μ, and thence fall off to become inappreciable with the last part of the red at .780μ.

The observations of Edridge-Green appear to fall in quite naturally with the experimental results herein presented, and seem to be in no sense opposed to the three-component theory coupled with reflex sensations.

[38] Nature, *107*, p. 826; 1921. Also Proc. Roy. Soc. B, *92*, p. 232; 1921.

The production by all colors of a reflex enhancement of the green-yellow part of the spectrum, no doubt accounts for the commonly observed fact that colors when diluted with white light tend towards yellow.

Since it is only the direct effects of color that have received such extensive investigation, it is not surprising that scores of color theories have been formulated. Probably most theories have in them some element of truth. But a solution of the "enigma of color vision" is not possible unless the whole effect of light waves on the complicated visual apparatus is known. The reflex sensations have never been taken account of, as hitherto their existence has only been suspected. To some extent one theory must be excepted from this statement. With such fragmentary facts as were known largely through the observations of McDougall himself, that investigator elaborated a theory[39] which, in a measure, recognizes the existence and function of reflex effects. He adopted the three-component theory, but adds, what from the standpoint of the present paper is unnecessary, a white sensation.

No theory at present formulated is sufficiently fundamental to include all the phenomena. But if the theory of three components, to which the names of Young and Helmholtz are attached, is enlarged by the inclusion of the reflex effects elucidated to some extent in this paper, that theory already so strongly grounded will afford explanations of and draw to its support many facts which have hitherto been unexplained, or even used against it. This statement must be qualified in one respect. Violet must be regarded as one of the fundamental color sensations in addition to red and green. For if these results speak with any decision at all, it is that blue as well as yellow are compound sensations.

The idea that the color sensations have developed out of a primitive white process receives no support from these experiments. There is no apparent development in any one direction more than in any other. But if there were development, it would rather be that white had been added to color and not color to white.

On referring to the reduced curves in Figs. 18 and 31, it will be seen that several of the equilibrium colors show vestiges of the three reflex depressions. Since the sensation curves are three in number it would appear to follow that absolute equilibrium would only occur when all three intersected in one point, or that one was zero in value when the

[39] Parsons, Loc. cit., p. 274.

other two intersected. As these conditions are never completely realized, perfect equilibrium seems to be impossible. Should more delicate methods of study of the visual reflexes be devised, as will probably be the case, this imperfection of equilibrium may then be more apparent than is the case with the persistency curves.

It is interesting to note that Burch[40] found the mean value of the transition point between red and green, for seventy persons, to be .573μ, and between green and blue to be .5003μ. The former is nearly the equilibrium color .570μ, and the latter is almost the mean of the group of three equilibrium colors, .480μ, .505μ, and .520μ.

It is quite possible that the sensory visual reflexes may also account for the intrinsic light of the eye. This is a general sensation on the retina of "very dark greyness." "That endogenous stimuli are the cause can scarcely be doubted, but whether these are primarily retinal or of central nervous origin remains uncertain, though it is now generally agreed that they are central."[41] Since the three sensations are always together affected reflexly the result will always be the production of white light of some intensity. This colorless light would necessarily be of central origin.

The determination of the reflex sensations of the color-blind will no doubt give valuable information concerning the nature of this peculiarity of vision. In a former research,[42] the author obtained the normal curves of about twenty-four subjects, including one totally color-blind, which were readily classified into six of the seven possible types of color blindness according to the Young-Helmholtz theory. These now need to be supplemented with the reflex and fatigue curves of similar types.

It is possible that the reflex effects in the color-blind, due to one or two or even all three sensations may be of equal magnitude with the direct effects and so produce white or grey in place of one or more of the color sensations. In total color blindness, vision may be achromatic because of constant equality between direct and reflex effects at all intensities, a condition which may also exist at moderate intensities in the peripheral retina in normal eyes. One fact only may be adduced at present. The explanation of the neutral point of the color blind may be the same as for complementary colors in normal eyes,—equality of direct and reflex sensations of *two* colors. The average position

[40] Parsons: Colour Vision, p. 115.
[41] Ibid: p. 104.
[42] Phys. Rev., *15*, p. 193; 1902.

of the neutral points determined by seventeen color blind subjects was .509μ, of which the greatest was .516μ and the least .495μ. This mean value lies between the two equilibrium points .505μ and .520μ, and is almost identical with one of the coincidence points, .501μ, of the fatigue and reflex curves.

V. EVIDENCE FOR SENSORY REFLEX

The author is not a physiologist and by no means holds the attitude perhaps implied by the brief, dogmatic title of this communication. It is obvious that physiology must decide whether these phenomena may properly be included under sensatory or sensory as distinguished from motor and secretory reflexes.

In support of this view, however, the following considerations are suggested.

The optic nerve of each eye proceeds to the optic chiasma where the decussation of the fibres from the nasal side of each retina takes place. From the optic chiasma, each optic tract proceeds backwards and ends in the lower visual centres on the corresponding side. From these centres, the corresponding radiations proceed to the occipital cortex of the cerebrum where the higher visual centres are situated. This constitutes the course of the afferent fibres from each retina to the occipital cortex.[43]

Arising from the cells of the occipital cortex are efferent[44] fibres which proceed via the optic radiations to end in the lower visual centres. From these centres new efferent fibres arise which travel to each retina.

The presence of both afferent and efferent fibres in conjunction with the retina of each eye, establishes the anatomical basis for the conduction of impulses from and to each retina. Up to the present no phenomenon has been observed which would demand the existence of efferent fibres to the retina from the higher centres. The fact, as is shown in this paper, that a color stimulus upon one retina enhances the perception of colors in the other retina indicates that there must be some route whereby the two eyes mutually affect each other to produce opposite effects. Due to the existence of the anatomical essentials of a reflex arc, i.e., the afferent and efferent fibres connected with each retina, it is possible to explain the effect observed by a

[43] Text-Book of Anatomy. Cunningham, 5th Ed. pp. 620 and 769.
[44] Cunningham, loc. cit., p. 620. Also: Gray, Anatomy p. 830; (1920 ed.).

reflex occurring which in some way changes the state of the receptor elements of these organs in the unstimulated retina. It is to be noted in the foregoing that the reflex observed from retina to retina is not of the same essential nature as the conventional motor or glandular reflexes, since in the first, the efferent impulses do not produce a motor or a secretory effect, but in some way alter the visual end organs so that they perceive certain colors in a different way. For this reason it seemed appropriate to call the new effect a sensatory or sensory reflex. This effect is not only transferred from one eye to the other but occurs also in the directly stimulated retina as well.

The experimental results set forth in this paper seem to be included in Howell's[45] definition of a reflex action, which is stated to mean "the involuntary production of activity in some peripheral tissue in consequence of a stimulation of afferent nerve fibres," the conversion of the afferent into the efferent impulse being effected in the nerve centres.

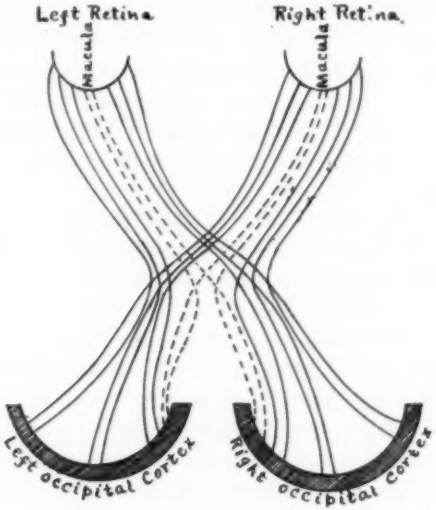


FIG. 35. *Optic Tracts.* (*Schaefer.*)

In Fig. 35, is reproduced Schaefer's very general sketch of the nervous tracts of the eye. In Starling's treatise on Human Physiology,[46] from which this figure is taken, Hartridge briefly describes the functions of the four classes of nerve fibres connected with the retina.

[45] Text-Book of Physiology, 8th Ed., p. 139.
[46] 3rd Ed., p. 552.

With three of these we are not at present concerned. But the fourth class is thus described: "The function of these inter-retinal fibres is not known definitely. It has been suggested that they cause changes in one retina when light falls on the other, as for example, cone movement . . . lastly binocular contrast and after images have been ascribed to them."

Hartridge does not describe the structure of these fibres sufficiently in detail to enable one to decide whether they are of such a nature as to constitute the reflex arc. But the proof that binocular contrast is due to the reflex color process, as shown above, may justify the ascription of this phenomenon to the fibres in question.

The possibility of the existence of a sensory reflex led Sherrington[47] to make extensive experimental search for it in the sensation of vision.

In further support of this view, that the effects observed are of a reflex nature, it is interesting to note that the latent period of this reflex, i.e., the time taken for the causative impulse upon the retina to produce its initial reflex effect, compares favourably with the latent periods of well known motor reflexes.

If it is assumed that the Bidwell effect previously described is due to a sensory reflex process, then from his description of the experiment, the appearance of the complementary color occurs in from about 0.015 to 0.02 second, which is the latent period.

The times which Jolly[48] obtained in estimating the latent period of the motor reflex known as the 'knee-jerk' in three cats are 0.0053 sec., 0.011 sec., and 0.012 sec. In these animals the spinal cord was severed so that the reflexes were free from cortical control, a condition which invariably decreases the latent period of a reflex. The order of magnitude of the periods of the sensory and motor reflexes is the same. From the arrangement of both afferent and efferent fibres it may be concluded that the sensory reflex is under the control of the higher centres, and therefore there is a legitimate reason for its period being somewhat longer than those measured by Jolly.

It may also be pointed out that the 'knee-jerk' is the most rapid reflex known,[49] and when it is found that the latent period for the sensory reflex compares favourably with that of the most rapid motor reflex there seems little doubt as to its nature.

[47] Integrative Action of the Nervous System. Lecture X, p. 354.

[48] Starling: loc. cit. p. 335.
For other reflex time determinations see Howell, loc. cit. p. 145.

[49] Starling: loc. cit. p. 335.

My colleague, Dr. C. H. O'Donoghue, Professor of Zoology, who very kindly read this last section, suggested that possibly the 'sensory reflex' may excite a secretory action connected with the production of the visual purple or other visual substances. This may be decided by ascertaining whether secretory action is sufficiently rapid to perform its function in the brief time limit allowable.[50]

If, however, the visual purple is to be regarded as a factor in reflex visual phenomena, another function will have been added to the list already suggested for this puzzling substance.

I desire to record my appreciation of the kindness of Messrs. A. Hollenberg, B.A., and M. S. Hollenberg, M.A., in bringing to my attention anatomical and physiological considerations intimately concerned with the experimental results of this enquiry, and to Mr. W. A. Anderson, B. Sc., for assistance in reducing the measurements.

I am especially grateful to the Honorary Advisory Council for Scientific and Industrial Research, Ottawa, for a substantial grant in aid of my researches in color vision.

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Ionization and Resonance Potentials of Argon and Krypton, and Excitation of the Krypton Spectrum.—Déjardin employs a conventional method and gives for these gases the values:

Potential	Argon	Krypton
Resonance	about 11.5	15.2
Ionization	15.2± 0.2	12.7 ±0.2
Double ionization	34.0± 0.5	28.25±0.5

The resonance potential is taken as the one at which ionization first appeared in a gas at about 1 mm pressure; the ionization potential as the one observed in more rarefied gas; the double ionization potential presumably is deduced from an inflection in the total and partial current curves. The data for argon are in excellent agreement with those recently offered by Horton and Davies. The data for krypton are the first to be published; it is pointed out that the ratio of double to single ionization potential is the same for both gases. The "red" spectrum of krypton appears as soon as some atoms are ionized and part of the "blue" spectrum as soon as some are doubly ionized, while other lines demand still higher voltages. In, this respect too, krypton behaves like argon. [G. Déjardin, C.R. *176*, pp. 894–897; 1923.]

K. K. DARROW

[50] See Starling, Loc. cit. p. 711.

DIFFUSE REFLECTION OF LIGHT FROM A MATT SURFACE

By Eugene M. Berry

Abstract

A theoretical discussion of certain hypotheses to explain diffuse reflection. That diffuse reflection of light from a matt surface can be explained by the hypothesis of countless small mirrors, (Bouguer's hypothesis) was denied by L. Grabowski. His objections to the hypothesis are not entirely valid. *Three ideal surfaces* are assumed and the laws of diffuse reflection from these are calculated and compared with experimental results. The agreement is only fair but perhaps as good as can be expected under the circumstances. The ideal surfaces are defined, statistically, and as the surface of ideal foam respectively.

A perfect matt surface is a surface which has no regular reflection and in which the reflection is independent of the azimuth. There are two general methods of attempting to explain the reflection from such a surface. The first, which was advanced by Bouguer[1] about 1750, is called the Bouguer hypothesis. In this, the surface is supposed to be made up of countless small mirrors, from which we have regular reflection. The other assumes that light penetrates a certain distance into the medium and that reflections occur at every point. The question discussed here is: can the Bouguer hypothesis explain diffuse reflection, or is it untenable? The latter alternative is maintained by L. Grabowski.[2]

He assumes the orientation of the mirrors is independent of the azimuth and finds the law of reflection, $f(\theta)$ which must hold for the small mirrors themselves, θ being the angle of incidence and of reflection for these mirrors. He finds that this reflection function must satisfy a certain functional equation, the only continuous solution of which is

$$f(\theta) = f(0)\sec^{2n}\theta. \qquad (1)$$

He says that except in the special case $n = 0$ or $f(\theta)$ equals a constant this solution is *a priori* impossible, for the reflection function must take the value 1 for $\theta = \frac{\pi}{2}$ and for $\theta < \frac{\pi}{2}$ must not be greater than 1; that

[1] Bouguer, Histoire de l'Académie Royale des sciences, Paris, 1757 (1762); and Traite d'optique sur la gradation de la lumière (Ouvrage posthume de M. Bouguer, et publié par M. l'Abbe de Lacaille), Paris, 1760.

[2] L. Grabowski, On the Theoretical Photometry of Diffuse Reflection, Astrophysical Journal, *39*, pp. 299–306; 1914.

is, the function must be an increasing function hence n must be positive; but if n is positive we have

$$\mathrm{Sec}^{2n}\theta = \infty \quad \text{for } \theta = \frac{\pi}{2}.$$

This is true but it should be observed that physical laws are in general only close approximations to the facts, and formulas frequently do fail in limiting cases. Fig. 1 shows a comparison between this

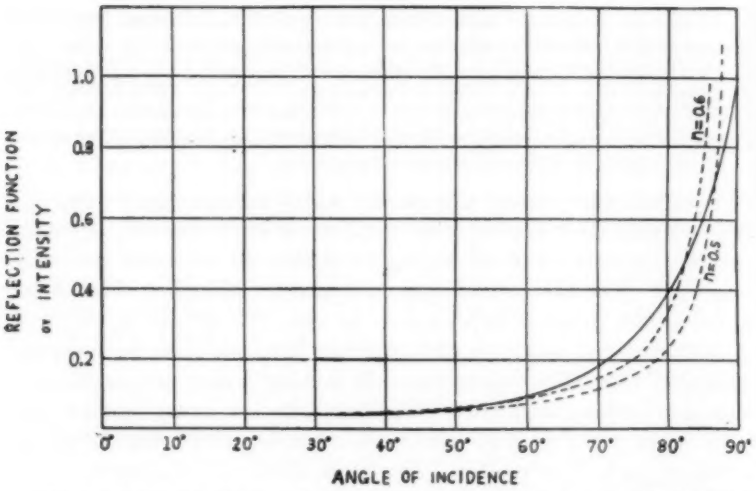

FIG. 1. *A comparison between Fresnel's reflection formula and a law of reflection which Grabowski finds must be satisfied by the small mirrors of a perfect matt surface. The full line is Fresnel's law for glass, the index of refraction being taken as* 1.5.

law and Fresnel's law to which it corresponds. The full line represents Fresnel's law for glass, the index of refraction being assumed to be 1.5. The dotted curves represent $f(\theta) = f(0)\ \sec^{2n}\theta$ for $n = 0.5$ and 0.6 respectively. Taking Fresnel's law as the standard we find that for $n = 0.6$ and $0° \leq \theta \leq 85°$ the error is less than 22% and for $n = 0.5$ and $0° \leq \theta \leq 60°$ less than 15%.

Lord Rayleigh[3] tested Fresnel's law for a few substances at very near perpendicular incidence. He says that a recently polished glass surface has a reflecting power differing not more than one or two percent from that given by Fresnel's formula, but that after some months or years the reflection may fall off from ten to thirty percent and

[3] Lord Rayleigh, On the intensity of light reflected from certain surfaces at nearly perpendicular incidence, Proc. Roy. Soc. *41*, pp. 275–294, 1886; Scientific Papers, *2*, pp. 522–542.

that without apparent tarnish. In consideration of this I would say the above agreement is as good as can be expected; especially when we remember that pure diffuse reflection is only approximated in any so-called matt surface and fails for grazing incidence, which is where formula (1) fails. Besides for metals Fresnel's theory has to be modified fundamentally. Hence the failure of formula (1) for grazing incidence does not appear to be a serious objection to Bouguer's hypothesis as Grabowski claims.

To show that diffuse reflection can be explained under the Bouguer hypothesis, it would be necessary to assume various ideal surfaces and investigate the laws of diffuse reflection from them and see how well they agree with experimental data. It is necessary to assume surfaces which can be handled mathematically. Among such surfaces are the following: 1st, ideally rough surfaces defined statistically in various ways; 2d, the surface of ideal foam which can be defined exactly. These cases are considered here.

T. K. Chinmayanandam[4] defines a surface to be "ideally rough" if the different portions of it are distributed at different depths according to the Gaussian probability law, but that tells us nothing about the distribution of the slopes so cannot be used in testing the Bouguer hypothesis.

We shall define a surface with the slopes distributed according to the Gaussian probability law. In an element of area, ds, of our ideal surface, let dA be the area of the mirrors whose normals lie within the solid angle $d\omega$. Then we may define our surface such that
Then from equation (2) we have,

$$dA = K\, e^{-\alpha p^2} dp \qquad (2)$$

where p is the slope of the normal to the mirrors and those mirrors whose normals lie in the solid angle $d\omega$ have slopes between p and $p+dp$.

For simplicity we shall only consider the case where the plane of incidence and of reflection is perpendicular to the plane of the surface. Let OP be an incident ray, OQ a reflected ray, OM the plane of the mirror, OS and OR normals to the mirror and to the surface respectively. Let θ and θ' be the angles of incidence and reflection respectively with respect to the surface, θ_1 and θ_1' for the small mirrors themselves, ψ the angle between the normal to the mirrors and the normal to the surface. Then we have

$$\psi = \tfrac{1}{2}(\theta - \theta'),\ \theta_1 = \theta_1' = \tfrac{1}{2}(\theta + \theta')$$

[4] T. K. Chinmayanandam, On the spectular Reflection from Rough Surfaces, Phys. Rev. Ser. 2, *13*, pp. 96–101; Feb., 1919.

As we are considering only parallel incidence, θ is a constant. Let $d\omega$ be the solid angle formed by a circular cone about OS as an axis and whose angle of opening is $d\theta_1$ then we have

$$d\omega = \frac{\pi(d\theta_1)^2}{4}$$

These mirrors will send their reflected rays into a solid angle $d\omega'$. The projection of $d\omega'$ on a unit sphere is approximately an ellipse whose major and minor axes are $2d\theta_1$ and $2\cos\theta_1 d\theta_1$ respectively. Hence

$$d\omega' = \pi\cos\theta_1(d\theta_1)^2$$
$$= 4\cos\theta_1 d\omega$$

Let C denote the luminous solid angular flux in any direction, and I the flux density or illumination on a large sphere of radius R, whose center is at the element of surface ds. Then $C = R^2 I$.

Assuming perfect reflection at the small mirrors and parallel incidence the illumination will have the same ratio to the flux density of the incident light as the effective area (that is, the area of cross-section of the incident beam, perpendicular to the beam, intercepted by the mirrors) has to the area illuminated on a large sphere of radius R. The effective area is $\cos\theta_1 dA$, the area illuminated is

$$R^2 d\omega' = 4R^2\cos\theta_1 d\omega$$

$$\therefore \quad I = K_1\frac{\cos\theta_1 dA}{4R^2\cos\theta_1 d\omega} = \frac{K_1 dA}{4R^2 d\omega}$$

Then from equation (2) we have,

$$I = \frac{KK_1 e^{-a^2p^2}dp}{4R^2 d\omega} \tag{3}$$

But $\quad p = \tan\psi = \tan\tfrac{1}{2}(\theta-\theta')$

and $\quad dp = -\tfrac{1}{2}\sec^2\tfrac{1}{2}(\theta-\theta')\,d\theta'$

and equation (3) becomes

$$I = -\frac{K\,K_1 d\theta'}{8\,R^2\,d\omega}\sec^2\tfrac{1}{2}(\theta-\theta')\cdot e^{-a^2\tan^2\frac{1}{2}(\theta-\theta')}$$

Then

$$C = C_0\sec^2\tfrac{1}{2}(\theta-\theta')\cdot e^{-a^2\tan^2\frac{1}{2}(\theta-\theta')} \tag{4}$$

where C_0 is the value of C for perpendicular incidence and reflection and here equals $-\dfrac{K\,K_1 d\,\theta'}{8\,R^2 d\omega}$ which is a constant from our assumptions.

The curve $y = K/(a^2+x^2)$ is shaped much like the probability curve $y = K\,e^{-a^2x^2}$, but does not approach the x-axis as rapidly near the ends as the latter. So instead of (2) we might assume

$$dA = \frac{K\,dp}{a^2+p^2}, \tag{5}$$

then we would get

$$C=\frac{C_0 a^2 \sec^2 \frac{1}{2}(\theta-\theta')}{a^2+\tan^2 \frac{1}{2}(\theta-\theta')}. \tag{6}$$

If a is taken as unity we get

$$dA=\frac{K\,dp}{1+p^2}=-\tfrac{1}{2}K\,d\theta',$$

which is the distribution of slopes on a sphere. This gives $C=C_0$.

Instead of comparing equations (4) and (6) with experimental results we shall use Lambert's cosine law for diffuse reflection, which is given by the equation,

$$C=C_0\,ds\cos\theta\cos\theta'. \tag{7}$$

Probably no surface ever diffuses light strictly according to this law but still it seems to be a fair approximation to experimental results. The factor $\cos\theta$ appears because the width of beam falling on ds is $\cos\theta\,ds$, and $\cos\theta'$ appears because the surface is supposed to appear equally bright, from whatever angle it is viewed.

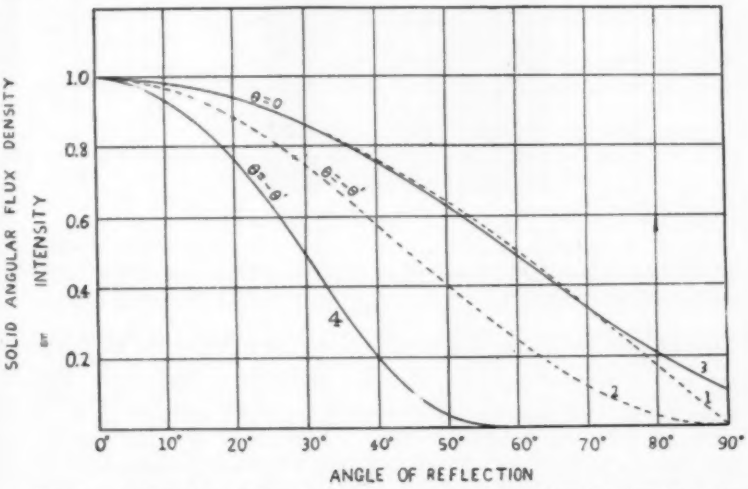

FIG. 2. *A comparison between Lambert's cosine law and the theoretical law for diffuse reflection from an ideal surface whose slopes are distributed according to the Gaussian probability law. The dotted curves are for Lambert's cosine law.*

Equations (4) and (6) can be compared with Lambert's law in special cases very easily. This is done in figures 2 and 3. Curves 1 and 2 represent Lambert's law for $\theta=0$ and $\theta=-\theta'$ (the reflected ray coincides with the incident ray) respectively. In Fig. 2, curves 3 and 4 represent

equation (4) for $\theta = 0$ and $\theta = -\theta'$ respectively with $a^2 = 3$. In Fig. 3, curves 3 and 4 represent equation (6) for $\theta = 0$ and $\theta = -\theta'$ respectively with $a^2 = 0.2$, 3′ and 4′ with $a^2 = 0.3$. Curves 1 and 3 in Fig. 2 agree very well; elsewhere the agreement is not very good. Thus our first ideal surface apparently has some specular reflection and reflects very little light back in the direction of the incident light, especially for large angles of incidence. The second one is much better in this respect.

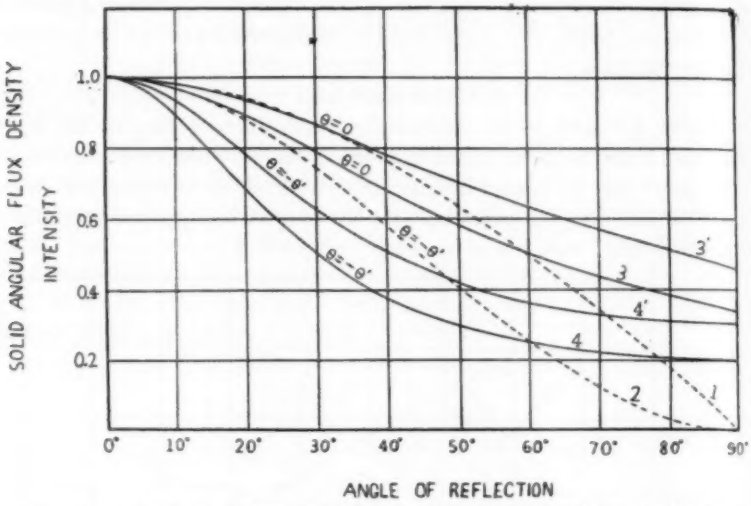

FIG. 3. *A comparison between Lambert's cosine law and the theoretical law for diffuse reflection from an ideal surface whose slopes are distributed according to a law which is somewhat similar to the Gaussian probability law. The dotted curves are for Lambert's cosine law.*

In the surface of ideal foam we assume the bubbles are all the same size and in the same plane, or what amounts to the same thing, a single layer of bubbles on a liquid surface. In this case each bubble is surrounded by six bubbles so that a horizontal cross-section of the layer will be in the form of a plane divided into equal hexagons. Each bubble will be separated from the rest by plane walls. The outside surfaces are portions of equal spherical surfaces, intersecting each other and the plane walls with dihedral angles of 120° each. The four lines made by the intersecting surfaces meet in a point, the angle between any two lines being $\cos^{-1}(1/3) = 109°\ 28'$. If the bubbles are not all the same size the surfaces could not be portions of spheres although nearly so if the bubbles are not too unequal in size.

In this case, assuming the spheres are infinitesimal in size, $C = C_0$ until we get to such an angle that the spherical surfaces interfere with the reflection from the adjacent spheres. Thus for perpendicular incidence $C = C_0$ for $0° \leq \theta' < 60°$ then would fall off rapidly and become zero when $\theta' = 70° 32'$.

A model of this surface has been constructed, using portions of spheres of one quarter inch radius, the surface being diamond shaped, seven by eight inches along the sides. The model was made of plaster of paris, then silver plated and polished to some extent. At a sufficient distance it appears much like a white matt surface. Fig. 4 shows a view of this model taken at a distance of about four feet.

FIG. 4. *Surface of ideal foam.*

In conclusion it seems that Grabowski's claim that the Bouguer hypothesis is untenable is not justified. Furthermore, it would appear that the hypothesis might give the laws of diffuse reflection as satisfactorily as the other hypotheses. To be sure the laws obtained for our ideal surfaces do not agree very well with observations on matt surfaces. However, this is not surprising. In the first place we have assumed the reflection at each little mirror to be independent of the actual angle of incidence on the mirrors. In our statistically defined surfaces no account is taken of the fact that the higher portions of the surface may shade some of the lower portions. Diffraction of light has been entirely neglected, which would be quite important for very small mirrors.

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The Generation and Utilization of Cold.—The December (1922) number of the Transactions of the Faraday Society contains the text of papers, together with the ensuing discussion, presented at the joint meeting of the Faraday Society and the British Cold Storage and Ice Association held in London, Oct. 16, 1922. The scope of the symposium can best be indicated by a list of the papers.

Part I. Laboratory Methods of Liquefaction.

On the Lowest Temperature yet Obtained.
By Prof. Dr. H. Kamerlingh-Onnes.

Apparatus and Methods in the Leiden Cryogenic Laboratory.
By Dr. C. A. Crommelin.

Ethyl Chloride. By Prof. C. F. Jenkin.

Part II. Industrial Methods of Liquefaction and Practical Applications of Low Temperatures.

A General Survey of the Subject. By K. S. Murray.

The Manufacture of Hydrogen by the Partial Liquefaction of Water, Gas, and Coke-Oven Gas.
By George Claude.

The Production of Liquid Oxygen for use on Aircraft.
By Edgar A. Griffiths.

The Heylandt Liquid Air Plant.
By A. J. Bremner.

Thermometric Lag with especial reference to Cold Storage Practice.
By Ezer Griffiths and J. H. Awbery.

Some Materials of Low Thermal Conductivity.
By Ezer Griffiths.

The Metallurgical Aspect of the Production of Cold.
By Cosmo Johns.

A Note on the Importance of the Study of the Crystal Structure and Properties of Metals at Low Temperatures.
By Cosmo Johns.

Of particular interest are the papers from the Leiden Laboratory. Dr. Onnes describes the method used in producing and measuring the lowest temperature yet obtained—"some hundredths of a degree below 0.°9 K." Dr. Crommenlin's paper, with the accompanying references, gives a description of the methods used at Leiden to produce, by means of various liquefied gases boiling under reduced pressures, any temperature below 0°*C* in order to make accurate physical measurements at those temperatures.

The several papers are copiously illustrated by sectional drawings of apparatus.

[Published by the Faraday Society, 10 Essex St., Strand, London, W. C. 2, Price 10 s. 6 d.]

F. K. RICHTMYER

INSTRUMENT SECTION

THE HILGER MICROSCOPE INTERFEROMETER

By F. Twyman

The microscope interferometer is an interferometer for measuring the aberrations of microscope objectives.

The Hilger Interferometers, designed by the writer, for the testing and correction of prisms, of lenses for axial pencils, and of camera lenses for both axial and oblique pencils, have been described (see Bibliography on p. 656), and it is desirable for those who desire to take up the use of the Hilger Microscope Interferometer to make themselves acquainted with references 3 and 6 (or 8, which is a reprint of 6).

Principles of Construction of the Microscope Interferometer

The Hilger Microscope Interferometer now to be described is a development of the Hilger Lens Interferometer*[3] which is shown in diagram in Fig. 1.

A collimated beam of monochromatic light is separated into two beams at the transmissively silvered surface of a plate of plane parallel glass K. The transmitted beam (which may be called briefly the test beam) passes through the lens T under test, and is reflected back from the surface of the convex mirror U, which is so disposed as nearly to coincide with the approximately spherical wave front of the light as it converges after passage through the lens T. Thus after reflection and passage back through the lens under test the wave front of the returning beam is approximately plane.

The second beam, which may be called the comparison beam, is reflected back along its own path by the mirror G, so that the two beams recombine at the silvered surface of the plate K. Portions of each beam then pass on together through the lens E which concentrates them on the eye of the observer situated at P. Under proper conditions (for which ref. 3 may be consulted) the observer then sees an interference pattern apparently located on the surface of the lens under test, and this pattern is a contour map, to a scale of half wave lengths, of the aberrations of wave surface occasioned by passage through the lens T of a plane wave.

(British patent No. 103832, 1916, and corresponding patents in other countries. Also additional patent applied for.)

* Reference numbers refer to citations at end of paper.

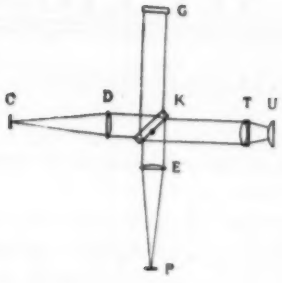

FIG.1

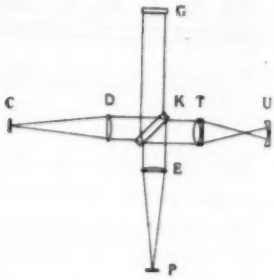

FIG.2

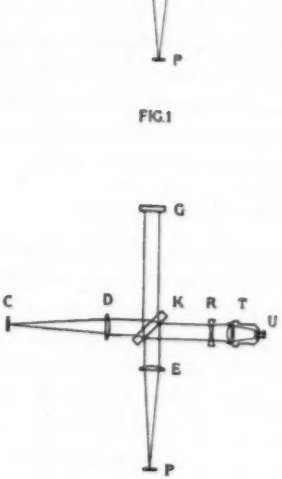

FIG.3

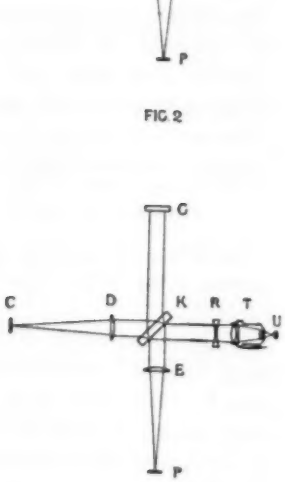

FIG.4

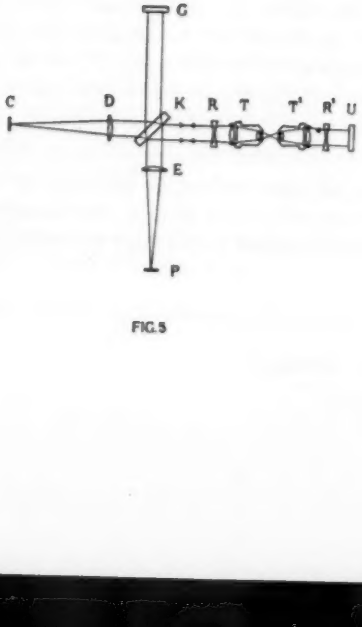

FIG.5

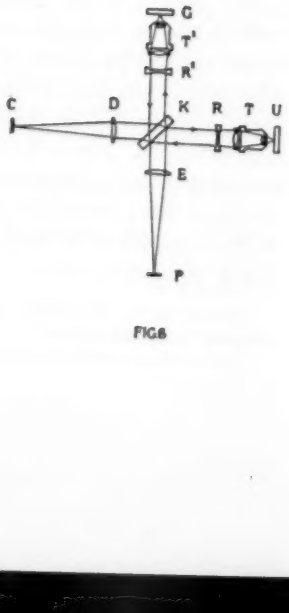

FIG.6

The arrangement may be modified by the use of a concave mirror instead of the convex one, as shown in Fig. 2.

If such an instrument be provided with sufficiently delicate adjustments for focusing the objective under test (namely, moving it to and from the mirror U), and moving the mirror U laterally, it becomes immediately suitable for exhibiting the aberrations possessed by a microscope objective when it is focused to produce its real image at infinity. The objective, of course, is represented by T in Figs. 1 and 2. If, as is customary, the objective is intended to produce a real image at a finite distance, such a condition is simulated by the introduction (see Figs. 3 and 4) of a negative lens R, corrected for spherical aberration and called, conformably to the nomenclature of the microscopist, the tube length lens. This lens must give to a parallel beam a divergence exactly corresponding to the convergence of the beam which obtains in the intended use of the objective.

The polishing of a concave mirror (Figs. 2 and 4) with sufficient precision not to introduce objectionable aberrations of its own becomes very difficult in the case of objectives of high numerical aperture, while if a convex mirror be attempted (Figs. 1, 3) it needs to be of very small size, and is on that account very difficult to produce accurately by ordinary procedure.

A small drop of mercury may be used as a convex mirror.[1] In this case also the observer has no direct way of assuring himself that the surface of the drop is sufficiently spherical and other modifications have therefore been devised, whereby the use of spherical mirrors is obviated. Figs. 5 and 6 show arrangements where the observer is satisfied if he can compare the objective to be tested with a second one (not necessarily identical) which he regards as a standard. In Fig. 5 the standard objective T^1 is in line with that under test, together with a second tube length lens R^1, while in Fig. 6 they are put in the comparison beam. In both cases flat mirrors suffice in both the test and comparison beams. It will be seen that in Fig. 6 each ray returns, not along its first course, but along another course axially symmetrical with the first. This has the disadvantage that only in the case of an objective whose aberrations are symmetrical does the interference pattern represent the aberration truly, the aberration of wave front shown at two axially symmetrical points (a and a^1, Fig. 7A) being in each case that due to the passage of a ray through a in one direction and a^1 in the other direction. Thus a

[1] See note (a) for method of producing.

distribution of aberration, due, for instance, to a local defect truly as shown in Fig. 7B would appear in the interference picture as in Fig. 7C. The arrangement in Fig. 6 has the advantage that the optical elements traversed by the test and the comparison beams being more nearly alike than in any of the other arrangements shown, it is possible, for reasons that will be referred to later, to get more illumination. This is true in an enhanced degree in the arrangement Fig. 8, where the two paths have been made equal by the compensating plate K^1 (identical with K). Where, then, it is desired to get photographic records of aberrations of a number of objectives of similar focal length, arrangement Fig. 8, supplemented by an ocular examination for non-symmetrical faults by arrangements 3, 4, or 5, has much to recommend it.

FIG. 7

Instead of the comparison objective T^1, tube length lens R^1 and mirror G in Fig. 6, a lens W of comparatively long focus (50 or 100 mm for instance) corrected for spherical aberration and with a flat mirror in its principal focus, can be used, as in Fig. 9.

Fig. 10 shows an arrangement in which no curved surfaces come into question other than those in the tube length lens and those of the objective under test. Here the lens and mirror W and G in Fig. 9 are replaced by the prism X in Fig. 10. X is a tetragonal or "corner cube" prism (or its equivalent) shown separately in Fig. 11.

Such a prism has the well-known property that every ray incident upon it is sent back parallel to its own path.[2] It has also the further property that the incident and reflected rays are precisely symmetrical with each other relatively to that ray which meets the apex of the prism. This property does not seem so well known, and a brief demonstration of it is given in note (b) of this paper.

It will be seen then why such a tetragonal prism can be used as in Fig. 10, in place of the object glass with mirror in its focus shown in Fig. 9.

[2] L. Silberstein. On Multiple Reflection. Phil. Mag. *32*, p. 487; 1916. Beck. Zeitschrift f. Instrumentenkunde, 1887, pp. 385–9. Grubb. English Patent 21856/1903. Zeiss. English Patent 5499/1906. Zeiss. English Patent 5500/1906.

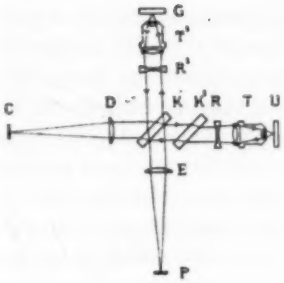

FIG. 8

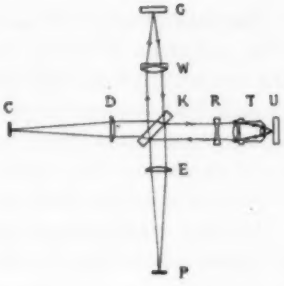

FIG. 9

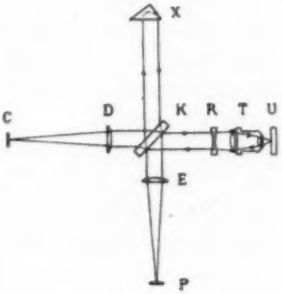

FIG. 10

FIG. 11

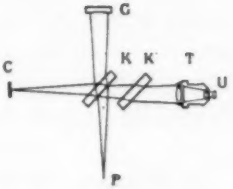

FIG. 12

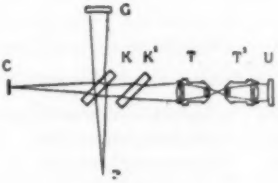

FIG. 13

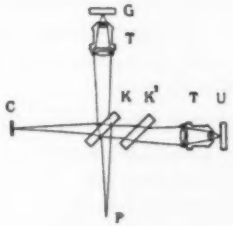

FIG. 14

The tetragonal prism has the disadvantage that it is difficult to make with sufficient accuracy, and that the edges of the right angles and the images of those edges divide the field into six parts by lines which, though fine, are by no means invisible, and thus slightly hinder the observation of the interference effects. On the other hand, the tetragonal prism has the advantage that it is somewhat easier to get into correct adjustment than are the corresponding lens arrangements.

Another arrangement which is of interest, although presenting no practical advantage, consists of the use of a very small tetragonal prism in place of the convex or concave reflecting mirror in Figs. 3 and 4.

ARRANGEMENTS OBVIATING THE NECESSITY OF THE NEGATIVE TUBE LENGTH LENS

It is possible to avoid the use of the negative tube length lens by placing the eye at the position where the objective forms in practice its real image. In this case, in order that the rays in the comparison beam may have a like convergence, a concave mirror is used where a plane one would otherwise be used, or some other device to cause the desired convergence. The lenses *D* and *E* are not required.

Some of the possible arrangements including this modification are shown in Figs. 12, 13, and 14, corresponding with those in Figs. 3, 5, and 8. Of all the above arrangements, the three which have been selected for the standard model instrument are those shown in Figs. 3, 9, and 10.

DETAILED DESCRIPTION OF THE APPARATUS

Light Source.—The light used must consist of a limited number of very homogeneous radiations. Table 1 gives radiations suitable for the purpose.

TABLE 1

	Wave length	*Source*
Mercury green	5461	Cooper-Hewitt mercury vapor lamp.
Mercury yellow	5791 5770	Cooper-Hewitt mercury vapor lamp.
Mercury violet	4359	Cooper-Hewitt mercury vapor lamp.
Hydrogen red (C)	6563	Hydrogen vacuum tube.
Hydrogen green (F)	4861	Hydrogen vacuum tube.
Hydrogen violet (G′)	4341	Hydrogen vacuum tube.
Neon yellow	5853	Special neon lamp.[3]

[3] Made for Adam Hilger Ltd. by the General Electric Company, England.

The use of several of these radiations permits the measurement of chromatic differences of focus to a very high degree of accuracy.

Apparatus.—A view of the apparatus is shown in Fig. 15.

The light from the source is reflected by an adjustable mirror through the condensing lens, by means of which it is condensed on the aperture of the diaphragm *C*.

The diverging beam of light is collimated by an achromatic lens *D*, and falls as a parallel beam on a plane parallel plate *K*, the second surface of which is silvered lightly so that a part of the light is transmitted and part reflected.[4] The major part should be reflected.

The mounting of the plane parallel plate is shown separately in Fig. 16.

The plane parallel plate *K* can be set in two alternative positions on the interferometer. The normal position is shown in Fig. 15. The second position, required only in adjusting the apparatus, is at 90° to the first, and requires the leg *L* to be pivoted round in order that it may find a bearing on the interferometer.

It will be noted that two milled head screws *H* and H^1 are provided for adjusting the inclination of the plate about horizontal and vertical axes. The pivoting axes are provided by two pairs of steel balls maintained in their positions by the single spring *J*, which also keeps the arm *A* against the points of the adjusting screws *H* and H^1.

One part of the light passes through the negative tube length lens *R* to the microscope objective and on to whatever reflecting device is in use mounted on the mechanical stage.

The microscope objective and the tube length lens are mounted on the same carriage (shown in Fig. 17) and are very carefully fixed so as to be co-axial.

The carriage moves along accurate steel ways under the action of a micrometer screw. This screw can be rotated directly by means of the handle *M* or through the intermediary of a slow movement drum *N*.

The greatest care is taken to ensure regularity and certainty in moving the objective longitudinally.

The micrometer screw has 1 mm pitch. The divided head attached to the screw has 100 divisions, and one turn of the slow motion screw, whose head is also divided into 100 parts, corresponds to one division on the main drumhead. Thus one division on the drumhead of the slow motion screw corresponds to 0.1μ (one ten thousandth of a millimeter).

[4] See Appendix, Note C, for a suitable method of partial silvering.

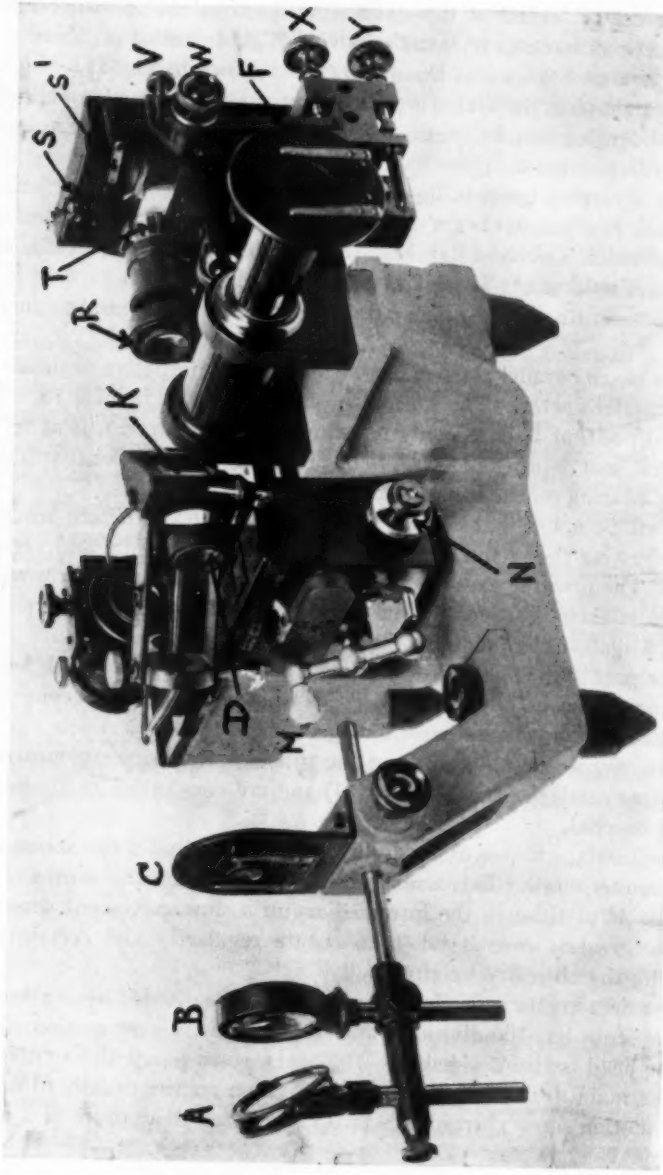

FIG. 15. *General view of the Microscope Interferometer.*

The slides are massive, and of steel,[5] and the sliding surfaces are worked *optically flat*.

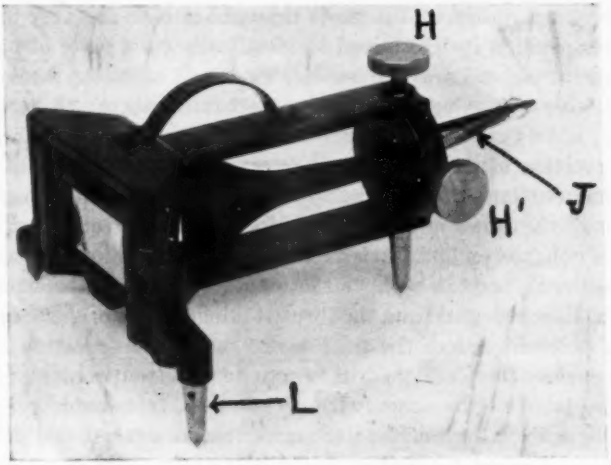

FIG. 16. *Mounting of plane parallel plate K.*

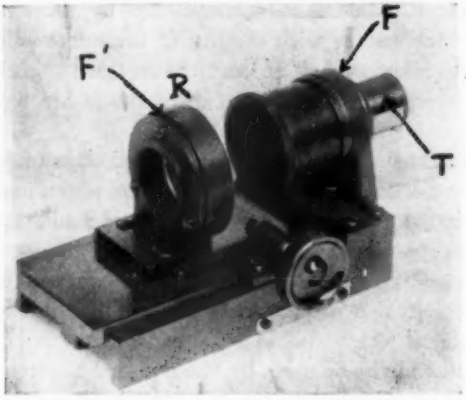

FIG. 17. *Objective Carriage.*

[5] The slides are made from forgings of specially selected steel, the preparation and treatment of which to avoid subsequent deformations are carried out by Sir W. G. Armstrong, Whitworth & Co., whose long experience in the construction of standard screw gauges enabled them to supply us with a material excellently well adapted for the purpose in view.

The slow motion mechanism has a cover, with a window for reading the main drumhead.

The micrometer screw is cut between dead centers and ground at constant temperature, and is made throughout with the very greatest care. Its position is determined longitudinally by a plate of polished corundum at the end near the handle, which end takes the thrust when the objective is moving towards the mechanical stage. A system of opposing screws enables this plate to be set up perpendicular to the axis of rotation of the micrometer screw, whose end is polished to an approximately flat surface. Thus by reflecting light in the direction of the axis of the screw through the thickness of the corundum block (which is polished on both surfaces to enable the end of the screw to be thus observed), one can observe the interference fringes produced between the light reflected from the thrust-taking surface of the corundum and the polished end of the micrometer screw. Observation in this manner enables the plate of corundum to be made perpendicular to the axis of rotation of the screw with great and ascertainable precision. The other end of the micrometer screw thrusts against the polished end of a fixed screw, by means of which one is enabled to reduce the longitudinal play of the micrometer screw to a minimum, without introducing any longitudinal compression. The carriage is pulled towards the thrust by two spiral springs. The two thrust blocks under the carriage, which engage with the nut of the micrometer screw, are so adjusted as to allow a small amount of lost time.

But for the two springs referred to the carriage can be removed by lifting it off its slide and as easily replaced.

The tube length lens (see Fig. 17) is mounted adjustably relative to the microscope objective by means of a rack and pinion, and a division in millimeters indicates the distance from the shoulder against which the objective screws to the position which would be occupied by the real image of a point formed by the objective when it is so focused that rays to the left of R are parallel. In deciding on the fineness of focusing adjustment which should be aimed at, it was borne in mind that the apparatus might be used, by the removal of some of its parts, as a high accuracy micrograph for the use of ultraviolet light, and provision was made for an accuracy suitable should a microscope objective of 5 mm focus and numerical aperture 1.0 at tube length 160 mm operate with light of wave length 1862 A. Such an objective should resolve a separation of .000093 mm corresponding to a focusing adjustment of the objective of .00013 mm.

With regard to the precision with which a definite movement of the objective should be achievable by reading the scales of the drumheads it was considered desirable that the above accuracy should be attained in movements of 2 or 3 mm. It has been found in practice that one can move the microscope objective a definite desired amount of 2 or 3 mm and return to the same position with a precision of 0.1μ or less even if during the movement the slow movement be thrown out of gear and replaced again for the final adjustment.

Mechanical Stage.—The mechanical stage has the usual adjustments in two directions at right angles, but exceptional care has been taken to ensure that the movement should be strictly in one plane perpendicular to the run of the ways. Furthermore, the accuracy of adjustment aimed at is much greater than is usually considered sufficient in the microscope. The movement is produced by screws of 0.5 mm pitch which thrust against hardened and polished steel plates; one of these screws can be seen at *W* Fig. 15. No dovetailed slides are used, the two sliding plates merely bearing in each case against flat surfaces, against which they are maintained by springs (*S*, S^1, Fig. 15).

If required, a very fine lateral adjustment can be obtained by levers (one shows at *F*, Fig. 15) which clamp on the screws, these levers being in turn actuated by the fine motion screw heads *X* and *Y*. It is not necessary to have these fine adjustments when using radiations in the visible region.

Where the instrument is intended for conversion into a micrograph any standard form condenser can be mounted below the mechanical stage.

The adjustment of the mechanical stage is not intended to have a range of more than about 1.5 mm from the middle position.

Comparison Beam Carriage.—(see Fig. 18). This is so arranged that it will take the three forms of reflecting devices, shown in Figs. 3, 9, and 10, these devices being mounted in tubes which push into cylindrical fittings in the adjustable body, *B*, of the carriage. In removing a reflecting device, one has to draw it out by means of the milled ring *M*, while holding the ring *N* in position with the thumb and first finger of the other hand. The middle finger of the latter hand should rest on *B* to steady the carriage.

Two kinds of adjustment are provided, viz., tilting adjustments in azimuth and altitude by the screws *H* and *I* and lateral movements horizontally and vertically by means of the two screws *J* and *K*.

The tilting movement is required for the arrangement shown in Fig. 3: both the tilting and lateral movements for the arrangement in Fig. 9: and the lateral movement alone for the tetragonal prism shown in Fig. 10.

The cylindrical sleeve into which these reflecting devices fit pivots upon two pairs of steel balls placed in the neighborhood of the ring *N* and since the lever is 9 cm from the pivoting point and the screws are of .5 mm pitch, the tilting movement is amply slow enough for convenient adjustment.

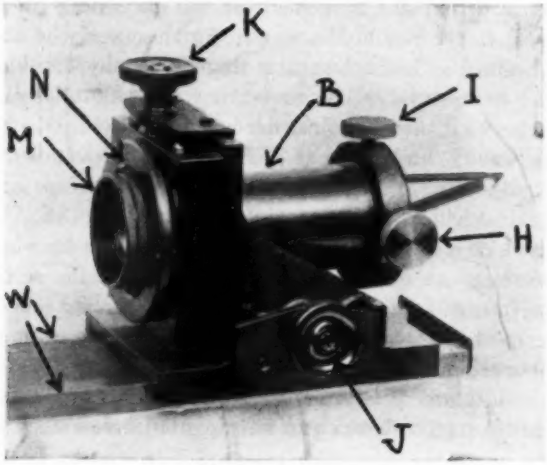

FIG. 18. *Comparison beam reflector carriage.*

The comparison beam carriage fits on ways *W* similar to those of the objective carriage, but no slow movement is required for its longitudinal adjustment. This can be accomplished by hand quite conveniently, and with sufficient precision, adjustment within a millimeter being amply sufficient to attain a position of good visibility of the bands.

MAKER'S ADJUSTMENTS

There are certain adjustments for which the makers must take responsibility but with which the observer should be acquainted. There are two reasons why this is desirable; first, that he may realize the need for them and be prepared to verify them if the instrument should sustain obvious damage or give dubious results; second, that he may know

that they have already received the maker's attention and should not be interfered with on a mere conjecture.

The Mechanical Stage.—This has been carefully squared on relatively to the ways so that not only is the face against which the microscope slide lies accurately perpendicular to the ways to about 3 minutes accuracy, but so also is the movement of the latter under the action of the mechanical stage adjusting screws.

The movement of the mechanical stage has been tested by mounting on the movable plate one of a pair of Fabry-Perot interferometer mirrors, the corresponding plate of the pair being mounted on the fixed portion of the stage. In this way it has been ascertained that the stage movement is uniform, in one plane, and sufficiently free from "lost time"; also that clamping the slow motion levers, to bring the fine adjustments into play, causes no disturbance of the slide.

The fineness of adjustment aimed at is such that if thus tested the bands of Hg 5461 can be set to any desired position to one tenth of a band (0.027μ). Such a fineness of adjustment exceeds all present requirements, but may be requisite if the interference instrument should eventually prove to be usable as a microscope interferometer with radiation as far as 1862 A.

Objective and Tube Length Lens Mounts and Carriage. The face F (Fig. 17) of the objective mount, and the face F^1 of the tube length lens mount are both made parallel to the face of the mechanical stage to about three minutes accuracy. The two mounts are made co-axial by screwing in place of the tube length lens cell a plug with a cylindrical fitting, and so adjusting the position of the two mounts that the end of the plug, which projects through the threaded objective mount, is central to it, to within about 0.05 mm.

The geometrical and optical axes of the tube length lens are made coincident to within about 0.025 mm.

Thus, if the objective itself be truly mounted, one is assured that objective and tube length lens are co-axial to within the limits indicated by the above figures.

The Collimator.—The collimator (consisting of the diaphragm C and the achromatic lens D, Fig. 15) requires to have its axis perpendicular to the face against which the microscope objective screws. Since this face has been made perpendicular to the run of the ways (see above) it suffices to set the axis of the collimator parallel to the run of the ways. This is done by setting up an accurate right angle prism resting on the top surface of the ways and perpendicular to the run

of the latter. If then light is allowed to fall on the diaphragm, a spot of light, which is an image of the diaphragm aperture, is formed on the diaphragm itself, the image being formed by the light reflected back from the surface of the right angle prism. The position either of the achromatic lens or of the diaphragm is then so adjusted that the back reflected spot falls on the aperture of the diaphragm itself. Simultaneously, the collimator is adjusted for focus, this being done by so adjusting the relative positions of diaphragm and achromatic lens that the back reflected spot is in focus simultaneously with the diaphragm aperture. Observation is made by looking in at the eye end of the interferometer (using an eyepiece) and with the diagonal mirror in the alternative position.

How To Set Up and Adjust the Microscope Objective

Arrangement of Fig. 3.—The method varies somewhat according to the particular arrangement which is in use. In Figs. 3 or 4 the plane mirror G is placed in the comparison beam carriage and the carriage brought to such a position that the distance from G to the selected point on the silvered surface of the diagonal mirror K is approximately equal to the distance from the same point to the mirror U. It is only when the path lengths in the test beam and the comparison beams are approximately equal that the interference bands are seen distinctly.[6] The aperture in the diaphragm C, Fig. 15, should be illuminated with suitable light. Placing the eyepiece in the fitting provided at P the spot of light reflected from the mirror G back on to the diaphragm at C will be clearly seen simultaneously with the hole in the diaphragm itself. By tilting the mirror G by means of the screws H and I Fig. 18, the image of the hole should be made to fall on the hole. Let us suppose that a mercury drop has been selected to act as a mirror (see note on page 653 for suitable method of preparing this), and that such a drop, on a microscope slide, has been placed in position on the mechanical stage.

If the eye be placed so as to look into the objective from the direction customarily occupied by the condenser, a small bright spot of light will be seen which is the image of the aperture in the diaphragm. Adjust the drop on the slide on the mechanical stage so that it obscures symmetrically this spot of light. Then, observing the drop through the telescope with eyepiece removed, perfect the adjustment by the screws

[6] For visibility of the bands see ref. 6, p. 188, and ref. 9.

of the mechanical stage. The drop having been got by these means as nearly central with the objective as possible, cut out the light from the comparison beam by placing a piece of card in front of the mirror *G*. Then it will be found on slowly focusing the objective by means of the handle *M* (Fig. 15) and observing with an eyepiece at *P*, that a small image of the diaphragm aperture will be seen by reflection at the surface of the drop. To be precise, as one focuses towards the mechanical stage four small circular spots come into focus in succession. They are:

(a) The image of the diaphragm aperture formed upon the surface of the mercury drop.

(b) The image of the aperture of the achromatic lens *D* formed upon the surface of the mercury drop.

(c) The image of the diaphragm aperture formed at the center of the mercury drop.

(d) The image of the aperture of the achromatic lens *D* formed at the center of the mercury drop.

It is image (c) which is the correct one.

This should be got in focus and central in the field. On removing the eyepiece and looking in at *P* with the naked eye, one will in general see a round field of green light, which represents the surface of the mirror *G*, and in the neighborhood of the center of this circular green field a bright green spot. The objective should now be re-focused; when the small bright spot will enlarge in size until it reaches a maximum diameter which corresponds with the useful aperture of the back lens of the microscope. At this position the interference pattern will be seen.

Arrangement of Fig. 8. It is necessary in the Fig. 8 arrangement that the beam passing through the achromatic lens reflector should be parallel to the axis of the lens. This lens and mirror arrangement will act even when it is tilted as a reflector which sends the rays back in a direction parallel to themselves. It is necessary, therefore, to have some independent assurance that the incident rays are parallel to the axis of the lens. This can be made sufficiently certain by first putting in its fitting mirror *G* (Fig. 3), and adjusting it so that the spot of light is reflected back by it on to the diaphragm, as described in the adjustment for Fig. 3 arrangement. If the achromatic lens and mirror reflector is then carefully put in place of the mirror, the condition in question will be fulfilled with sufficient exactness. The tilting adjustment, of course should not be used after the interchange is made.

On looking in at P one will observe an illuminated field due to the light from the comparison beam. If the objective is approximately in focus on the surface of the plane mirror, a small bright spot is seen, which can be enlarged to its maximum diameter by focusing the objective. When this position has been found, the reflector in the comparison beam must be adjusted laterally by means of the screws J and K, Fig. 18.

When the position has been found in which the returning rays in both the test and comparison beams return to the same point of the diagonal mirror, the interference pattern is seen, but unless one has some indication that this correct adjustment is being approached, the search for the correct position may be indefinitely prolonged.

For this reason two steel points, mounted on brass plates, are provided—the one to stand on the ways of the comparison beam carriage, the other to stand on the microscope objective carriage. Each of these points should be shifted sideways until its shadow is in line with the pointer itself and the eye placed at P. When this is done for both pointers, they will in general not appear to occupy the same position. The one in the comparison beam should then be shifted sideways until it lies in the same line as seen from the eyepiece and P, no eyepiece being used, as the image of the pointer in the test beam. If then the reflector mount is shifted horizontally and vertically by means of the screws J and K until the tips of the two pointers and those of their images all lie in the same straight line, the interference appearances will be seen. It is assumed of course that the reflector carriage has first been put in such a position that the length of path of the rays in the test and comparison beams is approximately the same.

Tetragonal Prism Arrangements.—The adjustment of the tetragonal prism reflector is very similar to that of the achromatic lens and mirror reflector. The carriage should be placed in such a position along the slide that the apex of the tetragonal prism is about the same distance from any selected point on the diagonal plane as is the plane glass reflector in the test beam. Strictly speaking, one should, in measuring these distances, take account of the thickness of glass traversed, but it is seldom necessary to do so since interference bands will be seen with a rough adjustment of distance, and once the bands are seen the shifting of the comparison beam carriage to the position where the bands are most distinct is a very simple matter. The tetragonal prism requires no tilting adjustment; all that is necessary is that it should be adjusted laterally with the two screws. Its correct position is such that the six

dividing lines by which its field is divided into six segments are symmetrically disposed relative to the effective aperture of the back lens of the microscope objective, thus the adjustment of the tetragonal prism is rather simpler than that of either of the other devices.

THE INTERFERENCE APPEARANCES AND THEIR INTERPRETATION

The interferometer here described is designed for examination of the aberrations of axial beams. Designs have been got out for testing oblique pencils, but the apparatus then necessarily becomes very complex and its construction is not yet complete. None the less, it will perhaps be interesting to give here a few observations with regard to all the five accepted types of aberration. The following remarks are reprinted from the paper read before the Optical Society (London) by the author on the 14th April, 1921 (ref. 6), and have reference to appearances seen on the Camera Lens Interferometer—they would be, however, equally applicable to a Microscope Interferometer such as here described but possessing modifications enabling the wave surface aberrations for oblique pencils also to be examined.

OBSERVATION OF THE ABERRATIONS

The apparatus measures the degree to which the wave front, impressed by the lens on light from a distant point source, differs from a spherical wave front. The indications are given in aberrations of wave front to a scale of wave lengths, the aberration shown being in every case twice that present in the once transmitted beam which normally forms the image of a distant point. The form in which the indications are presented is that of a series of interference fringes which are lines of equal aberration of wave front. Otherwise expressed, the interference fringes form a contour map of the wave front with reference to a sphere.

The recognized aberrations of a centered optical system are, chromatic aberration apart, of the following forms:

Spherical aberration in axis,
Coma,
Astigmatism,
Curvature of image,
Distortion,

and are characterised as follows, quoting E. T. Whittaker.[7]

[7] The Theory of Optical Instruments. Cambridge Univ. Press, 1907.

FIG. 19. *Interferograms and sections of wave fronts after single passage through lens.*

(a) *Spherical Aberration.*—"The marginal rays, or rays which pass through the outer zones of the lens, do not meet the axis in the same point as the paraxial rays."

(b) *Coma.*—"The linear magnification of a very small object, situated on the axis of the instrument, is different when different zones of the instrument are used to form the image."

(c) *Astigmatism.*—"A thin pencil which is not homocentric but diverges from (or converges to) two focal lines is said to be astigmatic."

(d) *Curvature of Field.*—"In order that a plane object may give a plane image the condition for flatness of field must be satisfied." Otherwise the focal surface is said to have curvature.

(e) *Distortion.*—"The image is said to be affected by distortion when the object (supposed to be a plane figure at right angles to the axis of the lens) gives rise to an image which is not geometrically similar to itself."

Drawings of interferograms corresponding with these aberrations appear in Fig. 19.

NOTES

(a) *Preparation of Mercury Drops.*—It is necessary that the drop should be small and free from grease and dust. Such drops may be prepared in the following way:—

Place a small amount (say .5 cc) of clean mercury in a test tube. Pour on it a few ccs of strong sulphuric acid in which as much chromic acid has been dissolved as it will take up. Shake for about 3 seconds, and then put under a water tap to rinse the acid away. Then rinse with distilled water; pour off as much of the water as possible.

Now with a pipette (the nozzle of which has itself been cleaned with the sulphuric and chromic acid mixture, washed and drained dry) transfer a drop of the mercury to a similarly cleaned microscope slide. Allow a second clean slide to fall on the first, when the mercury drop will be broken into a number of small ones suitable for use in the interferometer.

Another way (described by Barnard) is to hold a microscope slide over the mouth of a test tube in which mercury is boiling until drops of a suitable size (say .5 mm diameter) are formed by condensation.

(b) *Tetragonal Prisms.*—Two parallel rays in a homogeneous medium are the same distance apart after any number of reflections of the two rays at plane surfaces.

For in Fig. 20 let $A\ O\ B$, $A'\ O'\ B'$ represent two such rays each once reflected at a plane surface $E\ D\ F\ G$. If we imagine the whole system of rays rotated through 180° about an axis perpendicular to $E\ D\ F\ G$ through a point P midway between O and O', it is obvious that the so rotated system of rays will occupy the positions occupied by the rays before such rotation. Thus the distance apart of the parallel rays is the same after one plane reflection, and if the same after one reflection, then the same after any number.

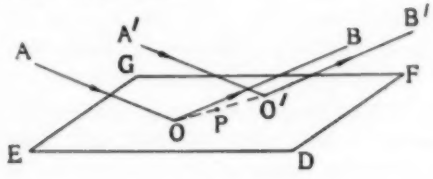

FIG. 20

It readily follows (see Fig. 21) that the incident ray which within the tetragonal prism has the direction KL will be symmetrical with the reflected ray PS relatively to the ideal apical ray MN (which we can also assume to have suffered three reflections and returned along its own path). Obviously the same is true concerning the corresponding rays FK, QM, ST, external to the prism, no matter how the latter be tilted.

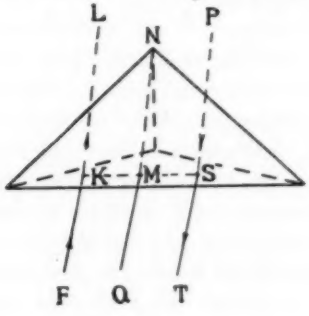

FIG. 21

(c) *The Deposition of a Transparent Film of Silver on Glass.—Solutions.* Prepare the following stock solutions:—

A.

10 per cent Silver Nitrate.

B.

40 per cent Formaline.

C.

Granulated Sugar	400 g
Alcohol	200 cc
Nitric Acid	10 cc

Make up with distilled water to 2000 cc. and allow to stand two weeks before using.

D.

Chromic Acid	250 g
Sulphuric Acid	1500 cc

Procedure.—The glass plate is placed in a glass dish, and cleaned first by strong nitric acid swabbed over the surface by small wads of cotton wool twisted round the end of a stick or glass rod, and then by allowing the glass to stand in some of *D* solution for 5 minutes. Pour off solution, rinse the plate thoroughly by a stream of running water, lifting the plate with a glass rod, to allow acid to escape from beneath the glass plate. Take 20 cc of *A* solution, add ammonia until the precipitate is *just* redissolved, add silver nitrate solution (any strength) until the liquid is a faint straw color. Make up to 100 cc with distilled water.

Reducing Solution.

5 cc of *B*
5 cc of *C* } mixed.

The mirror can be suspended with the face either upwards or downwards in the ammoniacal solution of silver. Then add the reducing solution. Keep the solution in constant motion until it becomes reddish in color. Pour off the solution and put in a second quantity of the ammoniacal solution of silver *without* any reducing solution. Allow the mirror to remain until the required density of deposit is obtained—this will take a few minutes only. The plates can then be rinsed with distilled water and dried. No polishing is required, the deposit being bright and uniform.

For the interferometer it is better to have the deposit of silver rather dense.

For the fully silvered mirrors any of the usual silvering processes may be used.

(*d*) *Microscope Tube Lengths.*—There are two tube lengths, 160 mm (continental) and 250 mm (English). The words "mechanical tube length" mean the distance from the shoulder of the objective to the tube end against which the eyepiece rim rests.

There is therefore no precision connection between back focus and tube lengths, but it is customary to correct for the position determined by the tube length and objective mounting.

In the microscope interferometer the negative "tube length" lens is mounted so that the distance from the objective shoulder to the back focus is from about 140 mm to 180 mm; or if a 250 mm tube length lens be supplied, an adjustment from about 230 mm to 270 mm is provided.

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LONDON, ENGLAND.

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Spectrum of the Aurora Borealis.—In a photographic study of the spectrum of the aurora between wave lengths 3135 and 6465 a large number of lines and bands of nitrogen were observed, no other known line, and four otherwise unknown lines, (one of them the celebrated green line). Since the relative brightnesses of the green line and the nitrogen lines are the same at different levels of the aurora, Vegard concludes that all the unknown lines are also lines of nitrogen, perhaps resulting from ionizations of higher degree than any observed in the laboratory. [L. Vegard, C.R. *176*, pp. 947–950; 1923.]

K. K. DARROW

DEMONSTRATING COLOR-MIXTURE

By M. Luckiesh

The demonstration of principles of color is interesting and instructive. The author has developed many demonstrations of light and color which have been described in various books and journals but one of the most fascinating to him which has not been described previously is presented in Fig. 2. In Fig. 1 is shown a simple means of demonstrating color-mixture[1] which has been used successfully before large

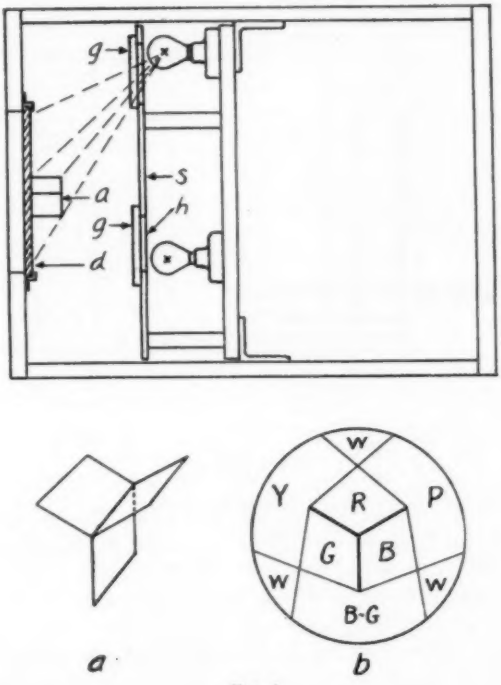

Fig. 1

audiences as well as in miniature. An object *a*, consisting of three opaque rectangles fastened together by a common edge and the pairs separated by 120 degrees, is glued perpendicularly upon an opal diffusing glass *d* (preferably flashed opal). Practically point sources of light are disposed as shown in Fig. 1 and in *a* Fig. 3 at the points of an

[1] Color and its Applications. 1915 and 1921, p. 66.

equilateral triangle. From these three light-sources red, green, and blue lights (primary colors) are obtained. Instead of using colored lamps it is more practicable to use colored glasses *g* or gelatine filters over holes *h* in an opaque partition *s*. When the dimensions are correct a colored pattern *b* Fig. 1 is seen on the opal glass *d* when viewed from the outside of the box. R, G, and B are the primary colors, the object *a* shielding in each case the opal glass from two of the primary lights. Y, P, and BG are the result of the mixtures respectively of each pair of primaries due to the fact that only one of the primary lights is shielded in each case. W indicates white light or the mixture of all

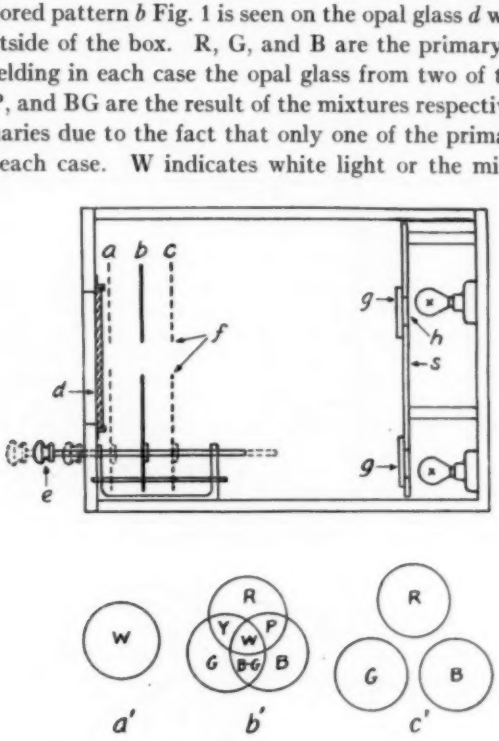

FIG. 2

three primary lights. Owing to the fact that the areas *W* are not equidistant from the three sources they are not exactly white or alike but partake slightly of the color of the nearest source. This demonstration on a large scale is effective and is quite attractive in miniature.

In Fig. 2 is shown a more striking method that has the additional fascination of movement. Here we also have an opal glass *d* and three primary colors obtained by means of three light-sources at the points of an equilateral triangle and behind colored glasses *g* which are over holes *h* in the opaque partition *s*. A metal sheet with a hole *f* can be

moved closer to or further away from the opal glass *d* by moving the handle *e* out and in. As a consequence of the different positions of the light-sources the image of the hole *f* is cast in different places on the opal glass *d* by the three light-sources. We therefore obtain in general three circular spots of light namely red, green, and blue. When the hole *f* is at the position *c* the three circles of primary colors are located with respect to each other as shown in c′. When *f* is drawn closer to the position *b* the overlapping of the three circles produces the familiar pattern *b′* where the center is white and the various primaries and their

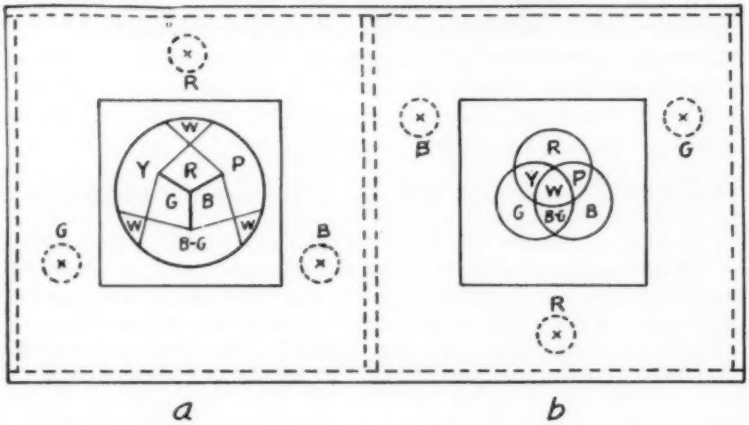

FIG. 3

complementaries are located as shown. When *f* is drawn into contact with the opal glass *d* we have the three colored circles in coincidence with a resultant circle of white as indicated in *a′*. This is one of the simplest and most striking methods of demonstrating the principles of color-mixture. The author has described elsewhere[2] other novel effects of colored shadows and images.

Inasmuch as practically point-sources of light are desired, it is convenient to use automobile headlight tungsten lamps for the smaller demonstrations. In this case it is practicable to combine both demonstrations in the same box using six 18-24 volt gasfilled tungsten headlight lamps in series. On 115 volts these will each use about 19 volts. For large demonstrations the gasfilled lamps of large size are fairly satisfactory as to size of light-source. Thin mirrors can be substituted

[2] Light and Shade and Their Applications, 1916, p. 52 et seq.

for the light-sources and these can be arranged around a common light-source, the images of the latter being used instead of the lamps as described in the foregoing.

Another demonstration which fits well with these is the color-triangle which the author has described elsewhere.[3] Light-sources of the primary colors—red, green, and blue—are placed behind and beyond the points of an equilateral triangle of roughed opal glass enclosed in a box. When optical conditions are correct a fair reproduction of the colored triangle is realized.

These and many other devices of this kind have been found very instructive and fortunately they are very easy to construct.

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Oscillating Mechanical Systems with Impulsive Excitation.—An oscillating mechanical system whose motion is maintained by the periodic application of a force does not swing with its natural frequency unless the motion and the applied force are out of phase by $\pi/2$. The writer investigates the effect of variation in the phase angle on the amplitude and frequency of self-excited mechanical oscillations.

When the damping is small, the impulsive actuating force may be considered as sinusoidal and the motion can be investigated analytically. In the case of strong damping, however, the equations become complicated and the problem can be more simply attacked graphically. The curves given show that, other things being constant, the amplitude is a maximum for a phase difference of $\pi/2$ and either an increase or decrease of this angle reduces the amplitude. The frequency on the other hand is increased when the impulse is advanced and decreased when it is retarded.

Quantitative experimental check of the theory is obtained in the case of a torsion pendulum and a ballistic galvanometer. [K. Kruger. Ann. der Physik. *4*, p. 291; 1923.]

L. BEHR

[3] Psych. Rev., *20*, May, 1913.

A SIMPLE DEVICE FOR RECORDING SOUND WAVES

By C. J. Lapp

This sensitive device was designed while research was being done on the wave form of the sound emitted by C. T. Knipp's singing tube. The principle used is similar to the one employed by D. C. Miller in his phonedeik. The optical reflecting system, however, is entirely different, being at the same time much simpler.

A diaphragm of dermatype paper is stretched over a two inch circular opening in a brass plate, 0.159 cm in thickness and held in place by a flat brass ring of the same thickness screwed to the plate To the center of the diaphragm are attached perpendicularly four or five silk fibers, the other ends of which are held by a very fine conical aperiodic spring. Across the diaphragm, 0.476 cm above and parallel to it, is very tightly stretched one strand of a three strand silk thread. This is passed 0.154 cm from the perpendicular fibres. An oscillograph mirror 0.0435 cm in width and 0.154 cm long is mounted with its plane parallel to the diaphragm between the horizontal silk strand and the vertical silk fibers. When a sound wave is caught by the diaphragm the mirror vibrates with it around the horizontal silk strand, causing a beam of light to vibrate and trace out the sound wave form on a moving photographic plate.

F. A. Shultze[1] has shown that paper is aperiodic and that a paper diaphragm is sensitive to sounds of any wave-length. The dermatype paper used is taken from dermatype stencils manufactured for the Edison-Dick mimeograph. This paper is flexible, and very strong.

In the diaphragm mounting the author has incorporated some new features which are shown in Figs. 1 and 2. No horn or resonating device of any sort is used in any of the work to increase the intensity of the sound brought to the diaphragm. Professor Foley of the University of Indiana read a paper before the St. Louis meeting of the American Physical Society in December, 1919, in which he clearly demonstrated the distorting effects of bent tubes and straight horns on sound waves. Although this diaphragm mounting was designed before Professor Foley's paper was read, the author was very careful in the designing to avoid air pockets of any sort.

[1] Annalen d. Physik IV, Folge, *24*, p. 785; 1907.

The inertia of the moving parts of the mounting is probably smaller than that of any diaphragm mounting heretofore used in sound wave analysis. The only masses involved are, the mass of three or four

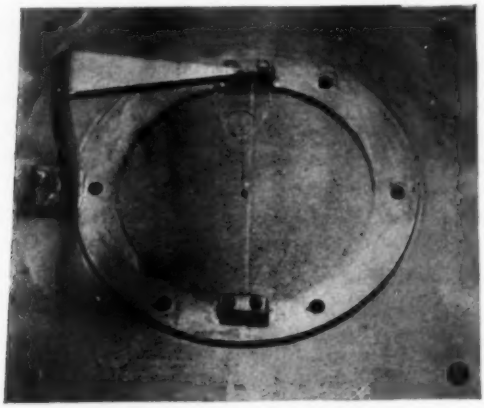

FIG. 1. *Diaphragm mounting.*

silk fibers 3 cm long, the mass of the mirror 0.154 cm long, 0.0435 cm wide and of microscopic thickness, and the mass of the small specks of shellac used to mount the mirror.

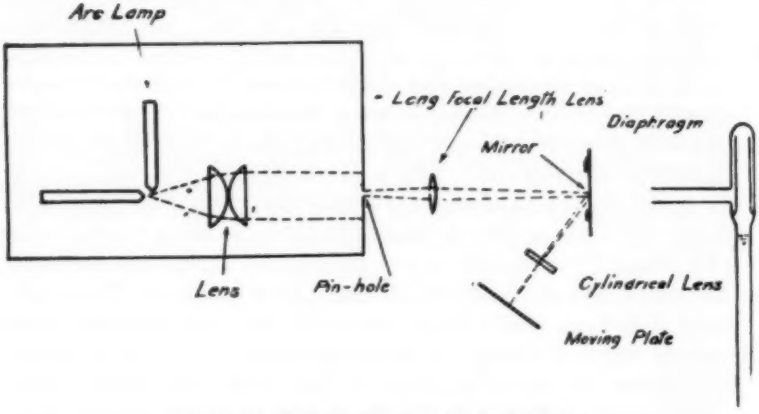

FIG. 2. *Apparatus arrangement for sound recording.*

The spring used is made by winding No. 40 steel wire on a brass cone of small dimensions. The period of any spring is a function of its diameter and the elasticity of the material used. The diameter of each

turn of wire is different from that of any other, therefore each turn of wire is different from that of any other, therefore each turn has a different period and the spring as a whole is aperiodic. While in use a small tuft of cotton is placed inside the spring to damp out sidewise vibrations.

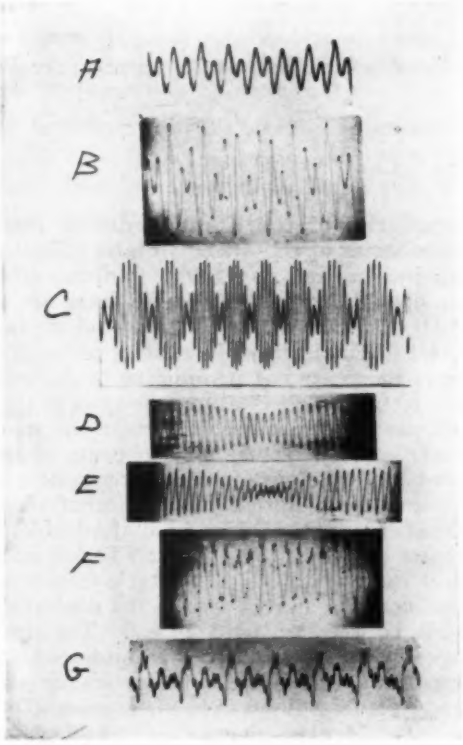

FIG. 3. *Recorded waves to show sensitivity of diaphragm.*

The other details of the set-up may be easily seen from the diagram, Fig. 2. This device was found to be very sensitive as is demonstrated by the photographs taken with it in Fig. 3. This is a series of curves taken on a falling photographic plate to show that the diaphragm was sensitive to even the faintest overtones. Wave A is the sound wave of an open organ pipe. Several overtones can be found in the

wave. Wave B represents the sound wave produced by two of Knipp's singing tubes sounding together, no resonator being used with either tube. This wave demonstrates the ease with which the dermatype diaphragm followed a complex wave form. Waves C, D, E, and F represent tones from organpipes and Knipp's singing tube sounding together. Wave G represents a note from a French horn. The wave forms here presented show clearly the sensitivity of the diaphragm.

The author wishes to acknowledge that this work was done at the University of Illinois while he was a student under Professor C. T. Knipp.

LABORATORY OF PHYSICS
STATE UNIVERSITY OF IOWA
MARCH, 1923

Relative Intensities of the Components of the Doublets in the Subordinate Series of the Alkali Metals.—The lines of the sharp series are emitted when electrons fall from various *s* orbits into the $2p$ orbit; they are double, in sodium, because there are two of the $2p$ orbits or energy levels. The relative brightness of the two components of a given doublet is a measure of the relative probabilities of an electron falling, from the given *s* orbit, into one or the other of the two alternative $2p$ orbits. Bartels measures the ratio of the intensities of the two components of each of five doublets of the sharp series, by a photographic method. Individual measurements of the ratio range between 1.92 and 2.17, (the lower frequency component is the brighter) and from all the material Bartels concludes that the ratio is 2 : 1, within the limits of experimental error, for all the doublets. An electron in one of the outer *s* orbits is twice as likely to fall into the upper as into the lower of the two $2p$ orbits. Dorgelo measures the ratio for two of the same doublets and for two of the doublets of the diffuse series; his data lie between 2 : 1 and 2 : 1.2. The absorption of the D-line frequencies by sodium vapor provides an interesting correspondence; when a quantum of one or the other frequency is absorbed by a sodium atom, an electron is lifted from the innermost *s* orbit into one or the other of the two $2p$ orbits; the relative absorbing power of sodium vapor for these two frequencies is a measure of the relative probability of an electron being lifted into one or the other of the alternative $2p$ orbits from the deeper lying *s* orbit. The ratio of the absorbing powers was measured some years ago, and found equal to 2 : 1, precisely as in these new observations. Dorgelo also measures the ratio for three doublets of potassium and one of caesium, and finds ratios of 2 : 1 to 2 : 1.4 with the former element, 6 : 1 with the later. [H. Bartels, Stuttgart; ZS. f. Physik, *14*, pp. 169–172; 1923. H. B. Dorgelo, Utrecht; ZS. f. Physik, *13*, pp. 206–210; 1923.]

K. K. DARROW

MULTIPLE RANGE POTENTIOMETERS

By Leo Behr

SYNOPSIS

The development of the potentiometer is briefly described, with special emphasis on the methods of checking the current against the standard cell and of changing the range.

Two designs for double range potentiometers are outlined which permit checking the current while operating on either range and in which the effect of variable factor plug contact resistance is completely eliminated.

Soon after the introduction of the potentiometer as a distinct unit, attempts were made to construct a convenient form of multiple range instrument. Raps[1] describes one with ranges of 11.0, 1.10 and 0.11 volts, the essential features of which are shown in Fig. 1. The Varley[2] Slide is used to obtain two dials while direct variation of the circuit resistance is employed in the final step. Dial I is made up of ten x1000 ohm coils, II of nine x1000 ohms, III of ten x10 ohms, IV of

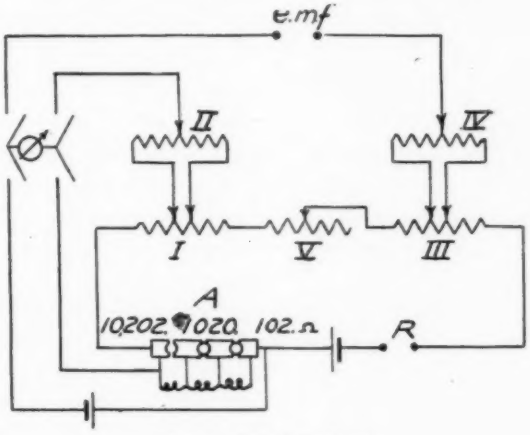

Fig. 1. *Raps Potentiometer.*

nine x10 ohms and V of nine x.1 ohm. With the aid of an external variable resistance (R_0) of 150,000 ohms connected to posts R, the current in the circuit can be adjusted to .0001, .001 or .01 ampere and the standard cell balanced against the corresponding coil in group A.

[1] Raps E. T. Z., *16*, p. 507; 1895, or Graetz, **Handbuch der Elekt. und Mag.** II, p. 208.
[2] Varley, Rep. of British Assoc., *36*, p. 14; 1866.

A high voltage may be measured by connecting it across R_0 and A in series, the former being adjusted until the drop across a suitable one of the coils in A balances the standard cell. The magnitude of the applied voltage can then be computed from the value of R_0.

Franke,[3] two years later, made a worth while advance in potentiometer design. Fig. 2, reproduced from his paper, shows the now familiar method of changing the range. The resistance of slide wire AB and each of the fifteen coils of dial K is four ohms. With the factor plug in the position shown, short circuiting W_3, the current is adjusted to .025 ampere by balancing standard cell N against the appropriate p.d. between brush K, and slider C; thus securing a range of 1.6 volts. With the factor plug in the other hole, coil W_2 is shunted around the

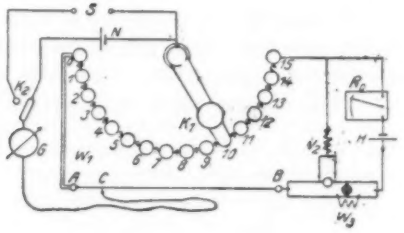

FIG. 2. *Franke Potentiometer.*

dial and slide wire reducing their resistance to one tenth of its previous value while W_3 is introduced into the circuit to keep the total resistance constant. In the instrument as actually constructed, this method is extended to secure ranges of 1.6, .16 and .016 volts.

An important advantage of Franke's method of securing several ranges lies in the fact that the constancy of the battery current makes possible a compact design for the regulating rheostat R_0. Furthermore the comparatively low resistance of the instrument renders feasible the employment of a galvanometer of sufficient sensitivity to realize the increased precision made available by the low scale arrangement. It should be noticed however that W_2 is approximately nine ohms for the .1 range and eight-tenths ohm for the .01 range so that a factor plug contact resistance of .001 ohm would cause an .01 per cent error in the first case and 0.1 per cent in the second. This possibility of a not readily detectable error may be conveniently and very materially reduced by locating W_3 inside the shunt as shown in Fig. 4.

[3] Franke, E. T. Z., *18*, p. 318, 1897.

In 1906, Leeds and Northrup[4] introduced their convenient method of checking against the standard cell which is illustrated in Fig. 3. In common with Franke's instrument, however, the potentiometer current can be adjusted only when the factor plug is in the high range.

This inconvenience may be overcome by the arrangement of Fig. 4, which is extensively employed in the construction of low range thermo-

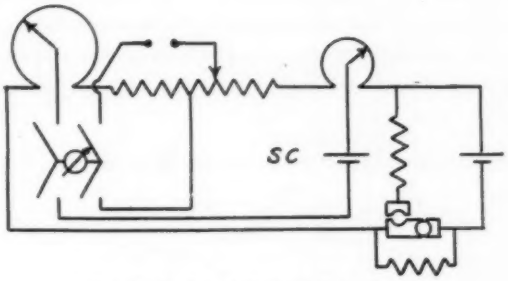

FIG. 3. *Leeds and Northrup Potentiometer.*

couple potentiometers, but for higher ranges labors under the disadvantage that the emf of the potentiometer battery must be equal to at least the sum of the maximum potentiometer voltage and the emf of the standard cell.

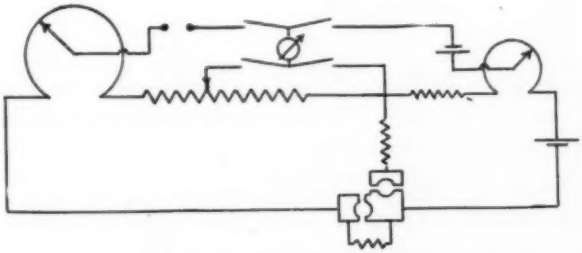

FIG. 4. *Low Range Potentiometer.*

Fig. 5 refers to a potentiometer with ranges of 1.6 and .16 volts which presents certain advantages over those already described. Slide wire *B* and each of the fifteen coils comprising *A* has a resistance of five ohms. This value has for convenience of description, been

[4] Leeds and Northrup, U. S. Pat. No. 819352; May, 1906.

assumed for *B* although it is ordinarily preferable to make the slide wire of somewhat higher resistance than a single coil in *A*. Branch *GD J* also has a resistance of eighty ohms. *HK* and *KG* are two equal coils the sum of whose resistances is nine times that of *H A B J;* in this case each coil therefore has a resistance of 360 ohms.

With the factor plug in the position drawn, 10/11 of the battery current flows through *A* and *B* and 1/11 by way of *HKGDJ*, so that with a battery current of .022 amperes, the current through the lower branch is .020 and through the upper branch .002 ampere. The p.d. across slide wire *B* and each coil of *A* is 0.1 volt and the drop from *K*

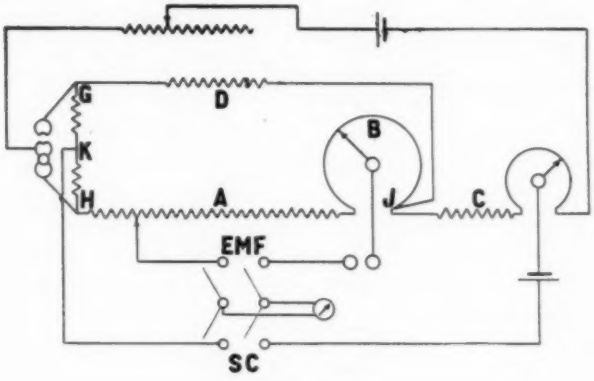

FIG. 5

to *J* is 0.88 volt. With the factor plug in the upper 0.1 position, the current through the lower branch is .002 and through the upper branch .020 amperes. The drop across each coil of *A* is now .01 volt but the p.d. from *K* to *J* is still 0.88 volt.

Using standard cells whose emf may vary from 1.018 to 1.020 volts, the resistance of *C* is computed to be $\frac{1.018-0.88}{.022}$ or 6.2727 ohms and the total resistance of the standard cell dial is similarly found to be 1/11 ohm. The current may now be checked against the standard cell on either range. The total p.d. across the instrument is 1.74 volt and it may therefore be operated on a single storage cell.

Fig. 6 is a diagram of connections for an instrument whose range is 0 to 20 and 0 to 200 millivolts. Slide wire *B* and each of the nineteen

coils of A has a resistance of .5 ohm. C is a ten ohm coil and D one of ninety ohms, with the factor plug in the position indicated and a battery current of .022 ampere, the p.d. across each coil of A is ten

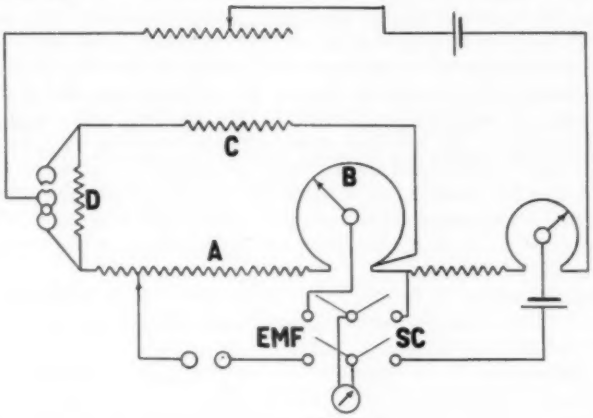

FIG. 6

millivolts, with the factor plug in the upper position the p.d. is one millivolt.

CORNELL UNIVERSITY,
ITHACA, NEW YORK.

Die Fernrohre und Entfernungsmesser. By A. König. Naturwissenschaftliche Monographien und Lehrbücher. Vol. 5. pp. 207; Julius Springer, Berlin, 1923.

This book is a well balanced and readable work possessing information of interest and value to the student, to the designer, and to the user of telescopes in general. To be sure, there may be some doubt as to the need for additional books treating this subject in a general way, but in this instance we find numerous items of interest—especially in connection with recent developments—that make the book a desirable addition to the works on the subject.

Except for the first chapter which deals with the formation of images in telescopes the treatment is essentially non-mathematical. The author has done well in limiting the extent of this chapter to a résumé of the laws of optical imagery in telescopes. One finds also a brief description of the eye and a discussion of its functions in vision, followed by a consideration of the phenomena of vision through a telescope, and then, a brief treatment of the aberrations without which no

book on practical optics seems to be complete. The few pages given over to the consideration of image formation from the standpoint of the wave theory form a welcome addition to the volume.

The next chapter presents details of construction of the telescope in many of its forms, both old and new. Unfortunately very few data are given and one wishes that more references had been included. On the other hand there is a generous collection of drawings and illustrations showing telescopes of all kinds. The descriptions are of a general nature but are sufficiently detailed for the student and casual reader, and suggestive to the scientist and to the instrument designer. Considerable space is allotted to military instruments, and here one finds mention and illustrations of instruments ranging from the small compact sighting telescopes to complicated periscopes and mast telescopes for observing, as from an elevation up to 26 meters, while comfortably seated near the ground.

In the discussion of astronomical telescopes—both refractor and reflector—we find many interesting points and good views of the world's largest and best telescopes.

The fundamentals of binocular vision are briefly presented. Here again the description of actual instruments occupies most of the space and views are given of instruments, old and new, from the peaceable little theater glass to the scissors telescope and the "hyposcope."

Methods of testing objectives and binocular instruments are allotted only six pages, of which one is a reproduction (without acknowledgment) of a plate from an English book.

We find next a chapter dealing with the simpler sighting devices—such as the "collimator"—and with the telescopic sights of Zeiss design. Among these are sights for rifles, for machine guns, and for artillery of all kinds. Usually the designing is of excellent grade. Some consideration is given in the discussion to the factors affecting the precision of aiming.

The short chapter on micrometers for measuring small angles is one of the features of the book that will appeal to those who may have occasion to design, construct, or use devices of this kind. To the average reader the word micrometer usually means a device for carrying a moveable cross-wire in a focal plane. Obviously, however, all those devices employed for the determination of small angles belong to this class. The heliometers of Bouger, Fraunhofer, and others, the sliding, tilting and rotating wedges, sliding lenses, and rotating parallel plates, all may be used to advantage in certain cases and are sometimes more suitable than a filar micrometer could possibly be.

Probably the outstanding feature of the book is the chapter on range finders. Not that it gives as much information on the different makes and types as may be found elsewhere—although recent German developments of special form are described—but the section devoted to the adjustment and testing of these sensitive instruments will be found interesting and instructive to anyone engaged in the study or manufacture of these instruments.

G. W. MOFFITT

AN IMPROVED BURETTE AND FILTER STAND

By Clifford D. Carpenter

In some recent experimental work in which it was necessary to make frequent and rapid titrations, it became evident that the ordinary burette stand could be made more convenient if the rod carrying the burettes could be easily raised and lowered. A slight alteration in the

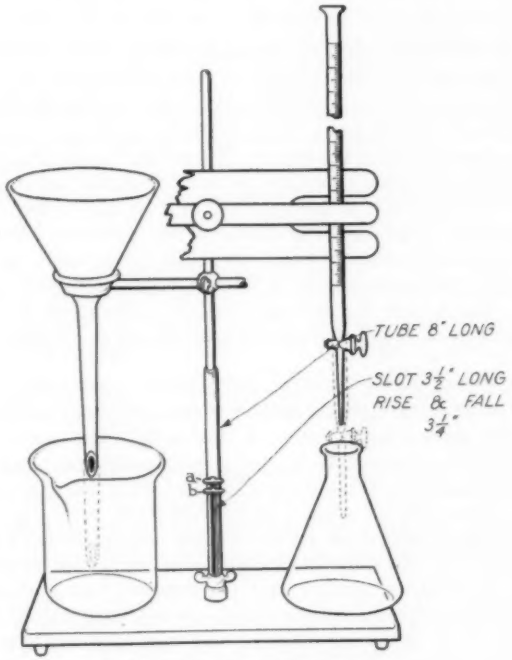

Fig. 1. *Burette and Filter Stand.*

present type makes it possible to put a flask or beaker under the burette without tipping. This is especially convenient when the beaker or flask is small. Such a stand gives the same advantages when used as a filter stand. The design of the proposed type and its operation is illustrated in the accompanying figure.

The rod that carries the support slides up and down in a tube which is fitted into the base. The raising and lowering is accomplished by a

small post (a) which is screwed into the lower end of the moveable rod. This post moves up and down in a slot in the tube. A second post (b) is fastened to the tube at the upper end of the slot and makes a rest for the thumb when the fore finger is under the moveable post in raising and lowering the rod. It has been found the best lengths are as follows—outer tube 8″ long, with a slot 3½″ long. The stands in use are porcelain base with brass rod and tubing.

COLUMBIA UNIVERSITY,
NEW YORK CITY.

General Astronomy. By H. Spencer Jones, Chief Assistant at the Royal Observatory, Greenwich. viii+392 pages, and 24 plates. Longmans Green & Co., 55 Fifth Ave., New York, $6.00.

A fascinating book on a fascinating subject, written "to appeal to the amateur, no less than the student." To write a book of this kind, sufficiently precise and comprehensive so that it should serve as an introduction to the student of astronomy, and at the same time tell an interesting story to the amateur reader, is no easy task. But Mr. Jones has succeeded admirably, partly by eliminating and subordinating mathematical treatment, partly by the copious use of well chosen diagrams. The text is further enriched by a series of 24 exceedingly fine photographs showing planets, solar phenomena, comets, star fields, nebula, etc.

After an introductory chapter on general considerations, devoted mainly to the celestial sphere, there are two chapters, dealing with the earth and its relation to the sun. Chapter IV is devoted to the moon and Chapter V to the sun. Worthy of special mention is the treatment of methods of measuring the solar parallax and the mass of the sun. Then, following a discussion of eclipses and occulations, come two interesting chapters on astronomical instruments and observations. Chapters IX and X, devoted to the planets and planetary motions, contain a large amount of descriptive matter and important data concerning each planet. Comets and meteors are discussed in Chapter XI. Interesting and valuable as is this description of the solar system, it is in the last three chapters, which are devoted to the stars and the stellar universe, where the reader finds the most fascinating story. Here one wonders whether to admire more the skill of the astronomer in first obtaining and then analyzing the immense mass of data on the basis of which to construct the present theories of Astronomy and Astrophysics, or the imagination which conceived these theores.

One must comment also on the excellent press work throughout the book. Taken altogether, the volume should appeal to a wide variety of readers.

F. K. RICHTMYER

METHODS FOR MACHINE CALCULATION OF RAYS THROUGH A LENS SYSTEM

By C. W. Woodworth

The following examples illustrate the substitution of simple arithmetical operations suitable for machine calculation in the place of the usual trigonometric formulas for lens calculation. The resulting methods are not only more simple but likewise more rapid than those hitherto used because of the elimination of unnecessary steps. They are of course based on exactly the same relationships always used in lens calculation and will require scarcely any explanation beyond giving the precise form of equation used at each step.

Three methods are shown below, one for edge or oblique rays, a corresponding method for paraxial rays, and the direct calculation of the power of the lens which is useful for checking the paraxial ray calculation.

The latter which will be given first consists of the determination of the power of each of the surfaces and of that due to each separation, all in diopters, millidiopters or inchdiopters, according to the units, meters, millimeters or inches, used for radii and separations, the sum of these determinations being the power of the lens. The method is precisely that given on pp. 243–244 of Vol. 4 of the Journal but with a different arrangement of the data, and the lens calculated is the one given on pp. 288–289 of the same number.

The optical data and the quantities obtained in the preliminary calculations may be indicated by capital letters and those of the final calculations by small letters.

Indices $A = 1.508721$ $B = 1.618812$

Radii $C = 259.52$ $D = 239.59$ $E = 40000$ inches

Separation of surfaces $F = 1.96$ $G = 8$ $H = .93$ inches

Diminished thicknesses $I = F/A = 1.299114$ $J = H/B = .574495$

Power of surfaces $K = (A-1)/C = .0019602$ $L = (B-1)/D =$
$.0025828-$ $M = (B-1)/E = .0000155$

Separation of centers $N = 2C - F = 517.08$ $O = C - D + G = 27.93$
$P = E - D - H = 39753.48$ inches

Sine of incident angle $Q = 18/C = .0693588$

Assumed paraxial angle $= .0000000693588$

CALCULATION OF POWER

$a=KI$		$b=aK$	$c=2K-b$
.0025465		.0000050	.0039154
$d=cG$	$e=a+d$	$f=eL$	$g=c-L+f$
.0313232	.0338697	.0000875	.0014201
$h=gJ$	$i=e-h$	$j=iM$	$k=g+M-j$
.0008158	.0330539	.0000005	.0014351

The quantity k is the power of the whole objective and $1/k=696.815$ inches is the focal length. Notice that the ratios of the successive focal lengths and separations are in the first column, the powers of the separations are in the third while the other two columns contain summations. The distance of the principal plane from the last surface is the quantity $i/k=24.172$.

CALCULATION OF PARAXIAL RAY

Q	$l=Q/A$	$m=Q-l$
.0693588	.0459719	.0233869
$n=1+mN/C$	$o=nA$	$p=m+o-n$
.0925691	.1396609	.0704787
$q=(oC-pO)/D$	$r=q/B$	$s=p+r-q$
.1430624	.0883749	.0157912
$t=(rD+sP)/E$	$u=tB$	$v=s+u-t$
.0162256	.0262662	.0258318

The principal focus is $CQ/v=18/v=696.815$ as before and every quantity in the preceding calculation can be figured directly from this, thus: $c=p/18$, $g=Bs/18$ and $k=v/18$. The focal distance from the last center is $Eu/v=40672.6$ and from the last surface is $E(u/v-1)=(t-s)\ E/18=672.6$. In the above calculation six cyphers after the decimal point are omitted in each term.

CALCULATION OF EDGE RAY

$Q=.0693588$	$l=.0459719$		$m=.0234243$
.9975918	.9989427		.9997256
$n=.0926436$	$o=.1397733$	$w=.0474380$	$p=.0708229$
.9956993	.9901835	.9988741	.9974890
$q=.1431440$	$r=.0884253$	$x=.0550686$	$s=.0157851$
.9897019	.9960828	.9984826	.9998754
$t=.0162198$	$u=.0262568$	$v=.0100391$	$v=.0258222$
.9998685	.9996552	.9999495	.9996664

The cosines must be determined in edge or oblique ray calculations and it is also more convenient to divide the last column making w the sine of the difference of the angles whose sines are n and o, x likewise for q and r, and y for t and u. Indicating the cosines by primes the equations for columns three and four are:

$$\begin{aligned} m &= Ql' - Q'l \\ m' &= Q'l' + Ql \end{aligned}$$

$$\begin{aligned} w &= on' - o'n & p &= wm' + w'm \\ w' &= o'n' + on & p' &= w'm' - wm \\ x &= qr' - q'r & s &= px' - p'x \\ x' &= q'r' + qr & s' &= p'x' + px \\ y &= ut' - u't & v &= sy' + s'y \\ y' &= u't' + ut & v' &= s'y' - sy \end{aligned}$$

The cosines of the first two columns are all determined by taking the square root of one minus the sine square. The other equations are the same as for paraxial rays. The first column are the sines and cosines of the incident angles, the second column of the angles of refraction, the third column of the difference between the preceeding angles, while the last column contains these functions of the successive angles between the ray and the optical axis. The focal distance from the last center is as before $Eu/v = 40673.2$ showing a spherical aberration of 0.6 inches. The sine condition is fulfilled when the significant figures of the quantity v in the two calculations are identical provided the digits of the initial axial angles are the same and the corresponding incident angles are used.

The special advantage of these methods lies in the fact that ordinary office help can be employed to make the calculations. At first one will need to furnish them the whole set of equations like those given above but as soon as they have learned the routine they will need only the data and the plus and minus signs of the first and last columns in the following form to calculate either the edge or paraxial ray.

				1.508721		−	
.0693588		.....				.....	
.....		.....				.....	
517.08	+	259.52				+	
.....		.....		.....		.....	
.....		.....		.....		.....	
27.93	−	239.59		1.618812		−	
.....		.....		.....		.....	
.....		.....		.....		.....	

39753.49 + 40000. +

. . . .

An Exhibit of Scientific Apparatus.—An exhibit of scientific instruments and apparatus was held at the Bureau of Standards April 20th and 21st in conjunction with the meetings of the American Physical Society and the Association of Scientific Apparatus Makers of the United States of America, held at the Bureau on the same days. The present exhibit was the second of its kind, the first having been held at the same place a year ago. The purpose of these exhibits is to show developments in American apparatus, and the present exhibit was striking evidence that encouraging progress in the design and manufacture of scientific apparatus is being made in this country. Exhibitors were principally members of the Association of Apparatus Makers. Among the apparatus exhibited mention may be made of the following:

The Precision Instrument and Thermometer Company of Philadelphia showed a representative group of laboratory and industrial thermometers and hydrometers. This company is the exclusive American manufacturer of pentane thermometers for low temperatures. One of its interesting instruments was a barometer mounted in gimbals for use on shipboard. This type of barometer has a cistern of fixed dimensions and a compensated scale which makes unnecessary an adjustment of the mercury level as in the Fortin type.

William Gaertner & Company of Chicago exhibited some optical instruments of recent design including a Nutting photometer, a wave length spectrometer, a Babinet compensator, an interferometer and a polariscope. In addition to these instruments a comparator consisting of a microscope in a 100 mm micrometer slide was shown.

An interesting exhibit was that of Marion Eppley of Newport, R. I. who showed his standard cadmium cells, and the apparatus used for purification of the various materials used in the construction of these cells. The Eppley Laboratory has developed a number of interesting devices for securing the extreme purity required in producing cells having greatest reproducibility and constancy of voltage.

Leeds & Northrup of Philadelphia had on display recently developed measuring instruments of considerable interest to those concerned with the testing of alternating current apparatus. One of these was the Silsbee transformer testing set for measuring the ratio and phase angle of current transformers. A portable phase shifter is used with this set. Other interesting instruments were an astatic dynamometer for use in measurements on power cables and for alternating current measurements in general, with an electromagnetic damping device; a galvanometer of high sensitiveness in which the entire circuit is of copper; and a model 7 Brooks deflection potentiometer. This company also had in operation an automatic potentiometer recorder connected to a Case photoelectric cell for recording daylight intensity. This photoelectric cell with a special light filter almost exactly reproduces the normal visibility curve.

Spencer Lens Company of Buffalo exhibited their Abbé refractometer; a research type of microscope with micrometer adjustments for slide mounting; and a student's spectrometer embodying excellent features of design. One of the noteworthy exhibits of this company was a compact microscope which may be folded into a case 4 x 2.5 x 1.1 inch, weighing 17 ounces. Its magnification range is from 40 to 350 diameters, and it is provided with a sub-stage condenser. Its ingenious features of design should make this a popular instrument with those requiring a microscope on field trips where lightness and compactness are of prime importance.

C. J. Tagliabue & Company of Brooklyn, specializing in instruments and apparatus for the testing of oils exhibited a variety of oil testing instruments as well as laboratory and industrial thermometers and hydrometers.

Bausch & Lomb Optical Company of Rochester displayed a group of optical instruments including a wave length spectrometer, a colorimeter, an Abbé refractometer, a binocular microscope, and a portable projection lantern. They also showed recent designs of a haemoglobinometer and a haemocytometer.

The exhibit of Central Scientific Company represented the developments of this company of the past year in the line of general physics apparatus. Among the new air pumps shown were a lecture table pump of the rotary type and a mercury vapor pump outfit of the diffusion type in which the high vacuum pump is constructed of Pyrex glass. The supporting pump is the Cenco Hyvac producing a vacuum of .001 mm. An electric heater with rheostat is used for heating the mercury vapor pump. Another piece which attracted attention was a large electromagnet of about 150 kg weight which, without water cooling, gives a field intensity of 16,000 maxwells per square centimeter between pole tips 15 mm in diameter and 10 mm apart. Another apparatus of new design was a lecture table rotator with all possible adjustments for lecture table use. Professor Sommerfeld's resonance spring for obtaining by simple experiment the elastic constants of the wire in the spring was also exhibited.

The C. H. Stoelting Company of Chicago displayed one of its kymographs (recording drum) driven by a clockwork motor. The feature of the drum is its very constant speed, the value of which can be changed within wide limits, and which is controlled by electromagnetic braking.